Introduction to GW from compact binary coalescence (CBC)

basic signal morphology & detection methods

Thomas Dent

IGFAE/University of Santiago de Compostela

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ICERM Workshop : Statistical Methods for the Detection, Classification, and Inference of Relativistic Objects

Plan of lecture

- Very brief introduction to GW
- Emission of GW from compact binaries
- Morphology and parameters of CBC signals
- GW detectors, response & noise
- The detection problem and matched filtering
- Signal geometry / template banks
- Challenges / frontiers (if time)

GW – a very brief introduction

- Weak field limit of GR

  Minkowski space

  $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1$$

  Valid at large distance from sources

- Physical content : Symmetric 2-index tensor
  Excitations travel @ speed of light
  Sourced by energy-mom. of 'matter'

  $$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

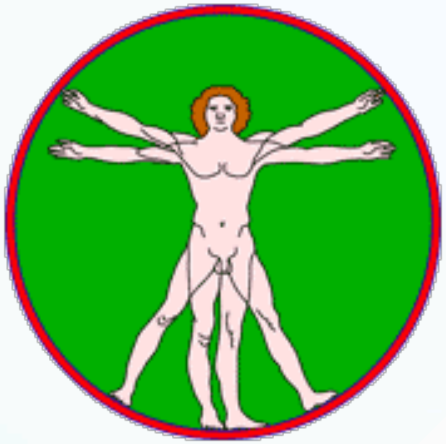
- In vacuo impose 'transverse traceless' condition
  Plane wave solution

  $$h_{ij}^{TT}(t,z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \cos\left(\omega(t - z/c)\right)$$

  2 independent
  pol. components

  $$h_+, h_\times$$

How to 'see' GW



- *Tidal* effect on spatially separated test particles
- Can extract energy (imagine a spring connecting particles)
- Measure variations in *distance* or *light travel time*

Strain $h(t) \sim \frac{\delta L(t)}{L}$

GW frequency : back–of–envelope

Gravitationally bound system, total mass *M*, size *R*

has a maximum *dynamical frequency*

$$R^2\omega_d^2 \sim \frac{GM}{R} \qquad\qquad \omega_d \sim \sqrt{\frac{GM}{R^3}} \sim (G\rho)^{1/2}$$

Sensitive frequency band of ground-based detectors

$10\ \text{Hz} < f_{\text{GW}} \sim \omega_{\text{d}}/\pi <$ few $\times\ 10^3$ Hz

Only *very dense* objects emit GW visible by LIGO

- MainSequence stars / planets : $\omega_{\text{d}} \sim 10^{-3} - 10^{-6}$ Hz
- WD : 0.1 – 10 Hz
- NS : 1000 – 2000 Hz
- BH : ?? $\quad (f_{gw})_{ISCO} \simeq 4.4\text{kHz}\left(\frac{M_\odot}{M}\right)$

GW amplitude : back-of-envelope

'Quadrupole formula'
strain at distance r from source
$$h(r) \sim \frac{1}{r}\frac{G}{c^4}\ddot{Q}$$

Q : quadrupole moment
$$Q \sim \int \mathrm{d}^3x\, x^2 \rho(x) \lesssim M R^2$$

(Maximum) rate of change described by dynamical frequency
$$\ddot{Q} \lesssim \omega_d^2 Q \sim \frac{GM^2}{R}$$

GW amplitude vs. compactness

- Order of magnitude bound on GW strain

$$h(r) \lesssim \frac{1}{r}\frac{G}{c^4}\frac{GM^2}{R} = \left(\frac{GM}{Rc^2}\right)\left(\frac{GM}{rc^2}\right)$$

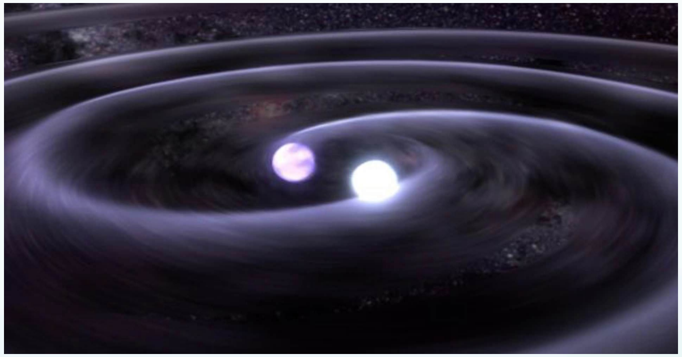
  Scales as M/R (not as ρ)

- Recall $R_S = 2\,GM/c^2$: $h(r) \lesssim \left(\frac{R_S}{R}\right)\left(\frac{GM}{rc^2}\right)$

  Object cannot be smaller than its own Schwarzschild radius (to avoid collapse into BH!)
  – 'Compactness' R_S/R strictly <1

GW are really small !

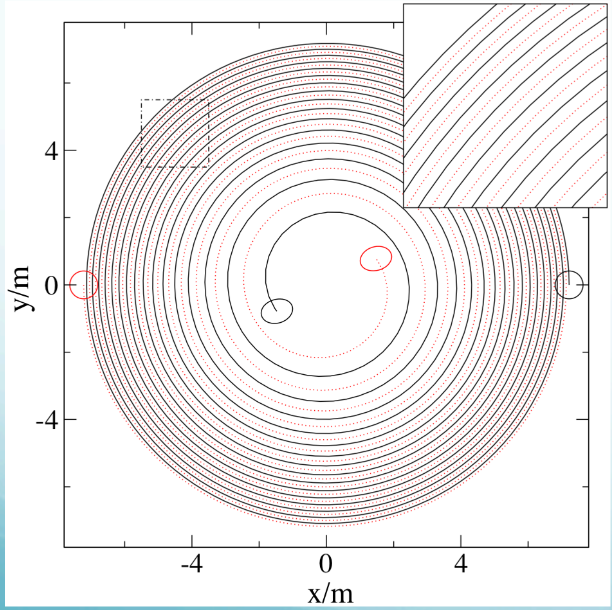
- Closest known NeutronStars $10^2 - 10^3$ pc away (Galaxy ~10^4 kpc)
- Most efficient GW emitters : *compact binaries* eg binary NS

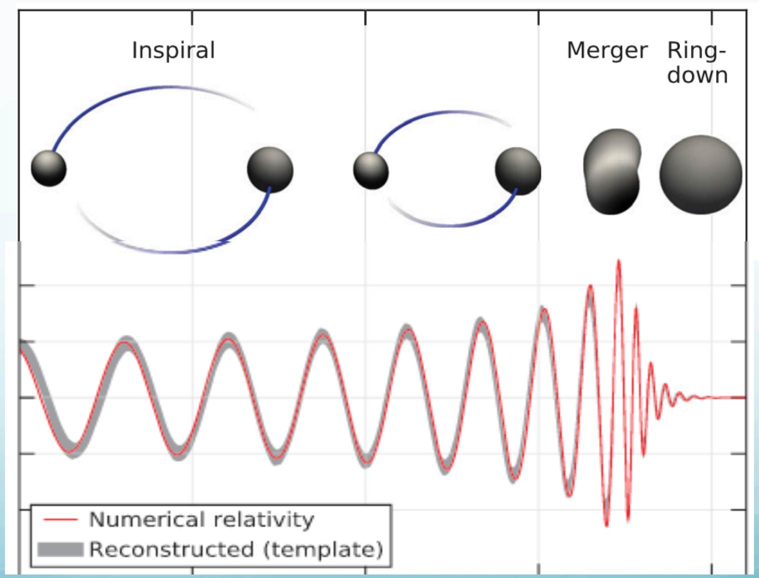


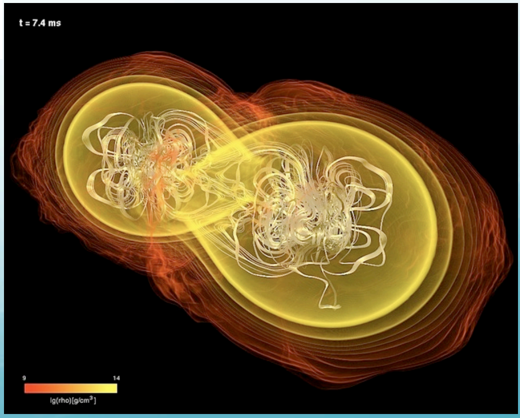
$$h(r) \approx 10^{-22} \left(\frac{M}{2.8\, M_\odot}\right)^{5/3} \left(\frac{0.01\,\mathrm{s}}{P}\right)^{2/3} \left(\frac{100\,\mathrm{Mpc}}{r}\right)$$

Compact binary mergers

- Binaries of NS / BH emit GW due to orbital motion
  - Orbit decays due to GW emission
  - Objects eventually collide / merge
  - Waveform predicted in GR given NS, BH masses/spins







GW emitted in circular orbit

- For emission in direction (θ , ϕ) find GW polarizations

$$h_+(t;\theta,\phi) = \frac{1}{r}\frac{4G\mu\omega_s^2R^2}{c^4}\frac{1+\cos^2\theta}{2}\cos(2\omega_s t_{\rm ret}+2\phi)$$
$$h_\times(t;\theta,\phi) = \frac{1}{r}\frac{4G\mu\omega_s^2R^2}{c^4}\cos\theta\sin(2\omega_s t_{\rm ret}+2\phi)$$

- GW frequency $\omega_{gw} = 2\omega_s$
- Amplitude grows with ω_s^2
- θ = angle between rotation axis and line of sight = inclination ι

Energy emitted as GW

- Power emitted in given direction:

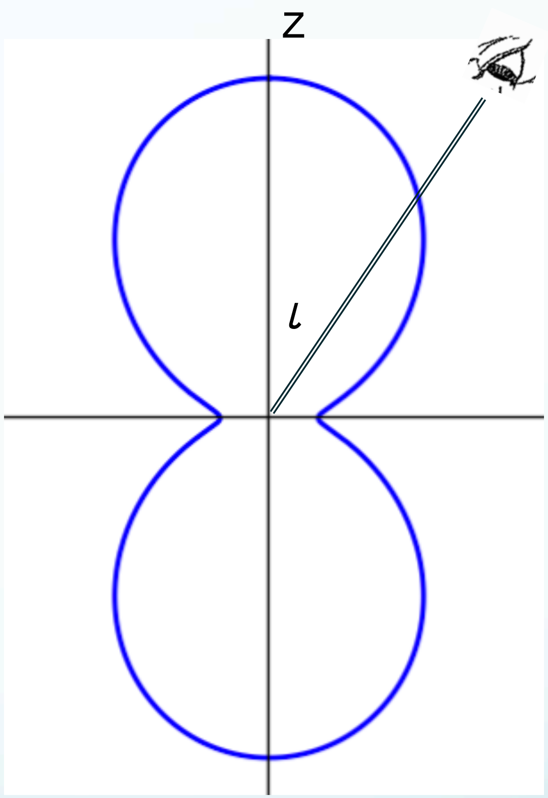
$$\frac{dP}{d\Omega}_{|\text{quad}} = \frac{r^2c^3}{16\pi G}\langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle$$

- ⟨...⟩ = average over few cycles : $\langle \cos^2 2\omega t \rangle = ½$
- Result:

$$\frac{dP}{d\Omega} = \frac{2G\mu^2R^4\omega_2^6}{\pi c^5}\left[\left(\frac{1+\cos^2\iota}{2}\right)^2 + \cos^2\iota\right]$$

Angular distribution of GW power

- ‘Peanut shaped’ emission along rotation axis



Integrate over dΩ: total power

$$P = \frac{dE_{\rm GW}}{dt} = \frac{32}{5}\frac{G\mu^2}{c^5}R^4\omega_s^6$$

Kepler's law and chirp mass

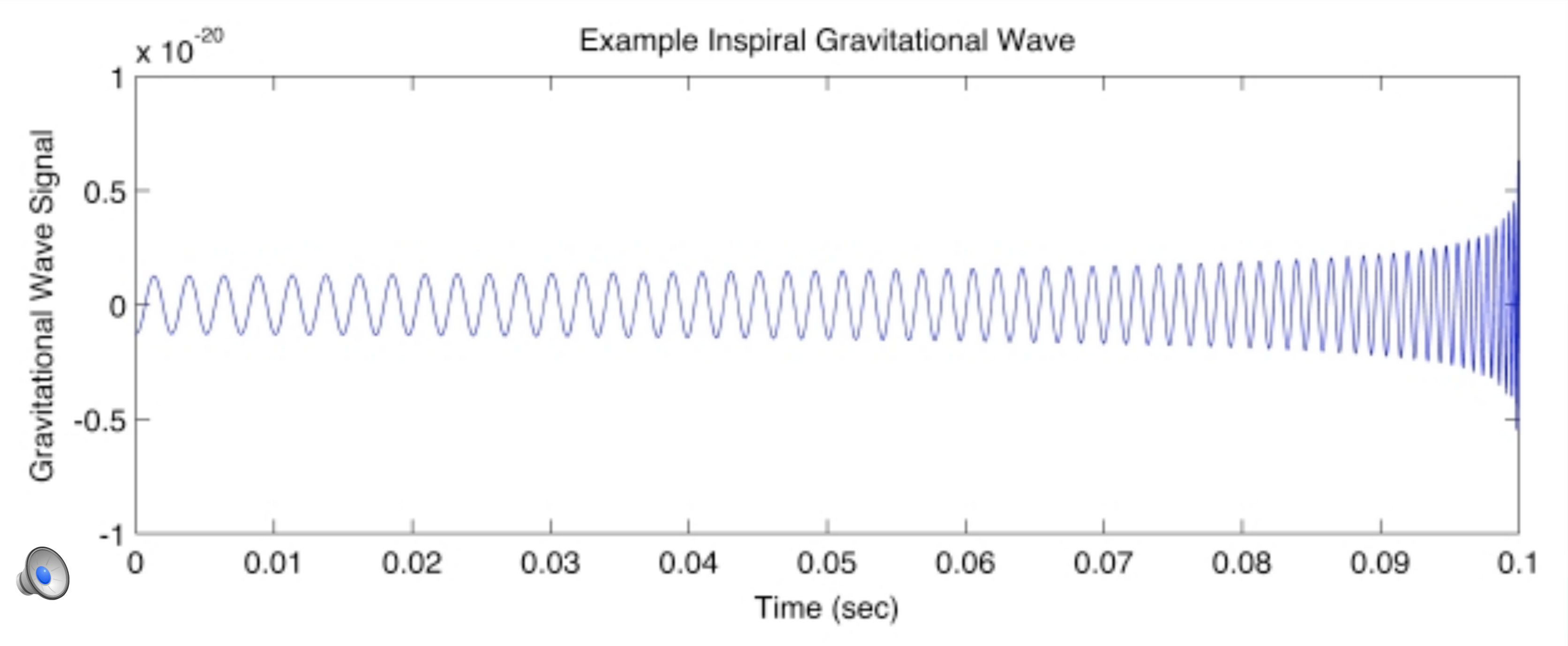
- Circular orbit: $$R = \left(\frac{Gm}{\omega_s^2}\right)^{1/3}$$
- Rewrite h_+ and P via 'chirp mass' $$M_c \equiv \mu^{3/5} m^{2/5}$$

$$h_+ \propto \left(\frac{GM_c}{c^2}\right)^{5/3} \left(\frac{\pi f_{\mathrm{gw}}}{c}\right)^{2/3}$$

$$P \propto \left(\frac{GM_c \pi f_{\mathrm{gw}}}{c^3}\right)^{10/3}$$

($f_{gw} = \omega_s/\pi$)

A binary inspiral chirp

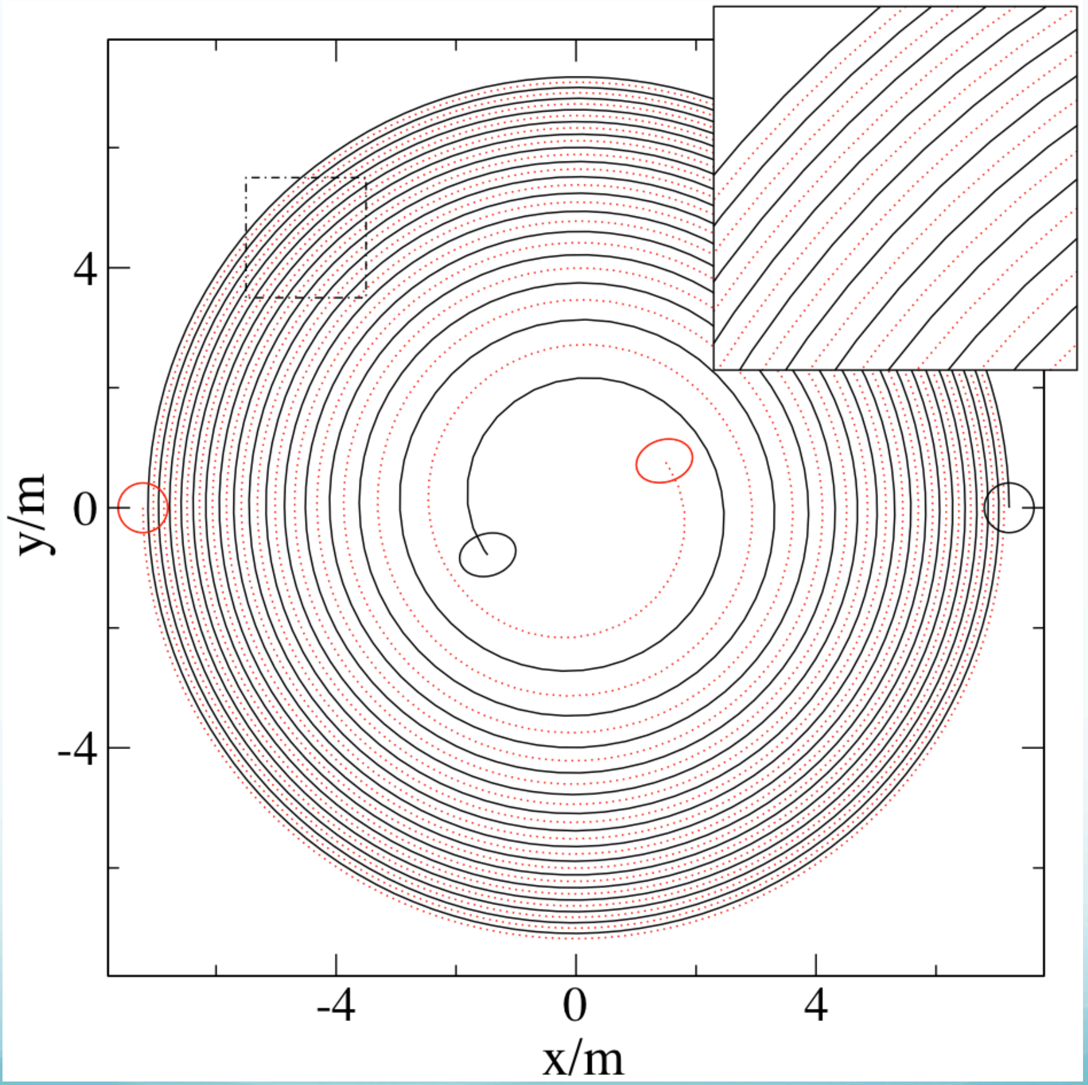


- Highest GW power in last few hundred cycles
- In LIGO frequency band if

  $m \sim$ few $M_\odot$ up to (few×10) $M_\odot$

Image: A. Stuver, LIGO

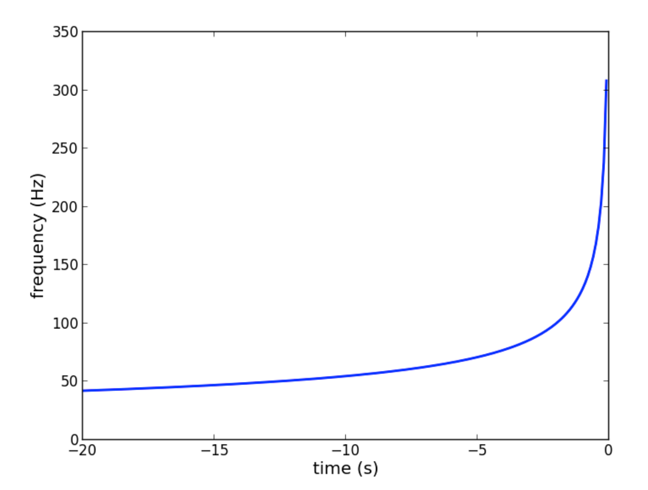
Binary inspiral orbit



Chirp in time domain

- Chirping frequency $f_{gw}(t)$ from loss of orbital energy via GW

$$f_{\rm gw} = 130\,{\rm Hz}\left(\frac{1.2\,M_\odot}{M_c}\right)^{5/8}\left(\frac{1\,{\rm s}}{\tau}\right)^{3/8}$$



$$h_+(t) = A(\tau)\frac{1+\cos\iota}{2}\cos(\Phi), \qquad \Phi(t) = \int dt'\omega_{\rm gw}(t')$$

$$A(\tau) \propto \tau^{-1/4}, \qquad \Phi = -2\left(\frac{5GM_c}{c^3}\right)^{-5/8}\tau^{5/8} + \Phi_0$$

$\Phi(t)$: 'gravitational wave phase'

Chirp in frequency domain

- Fourier transform $h_+(t)$ (not entirely straightforward!)

$$\tilde{h}_+(f) \propto e^{i\Psi_+(f)} \frac{1}{f^{7/6}}$$

GW phase in frequency domain

$$\Psi_+(f) = 2\pi f(t_c + r/c) - \Phi_0 - \frac{\pi}{4} + \frac{3}{4}\left(\frac{GM_c}{c^3} \cdot 8\pi f\right)^{-5/3} + \cdots$$

Higher terms in f $\propto$ v/c : 'Post-Newtonian' theory

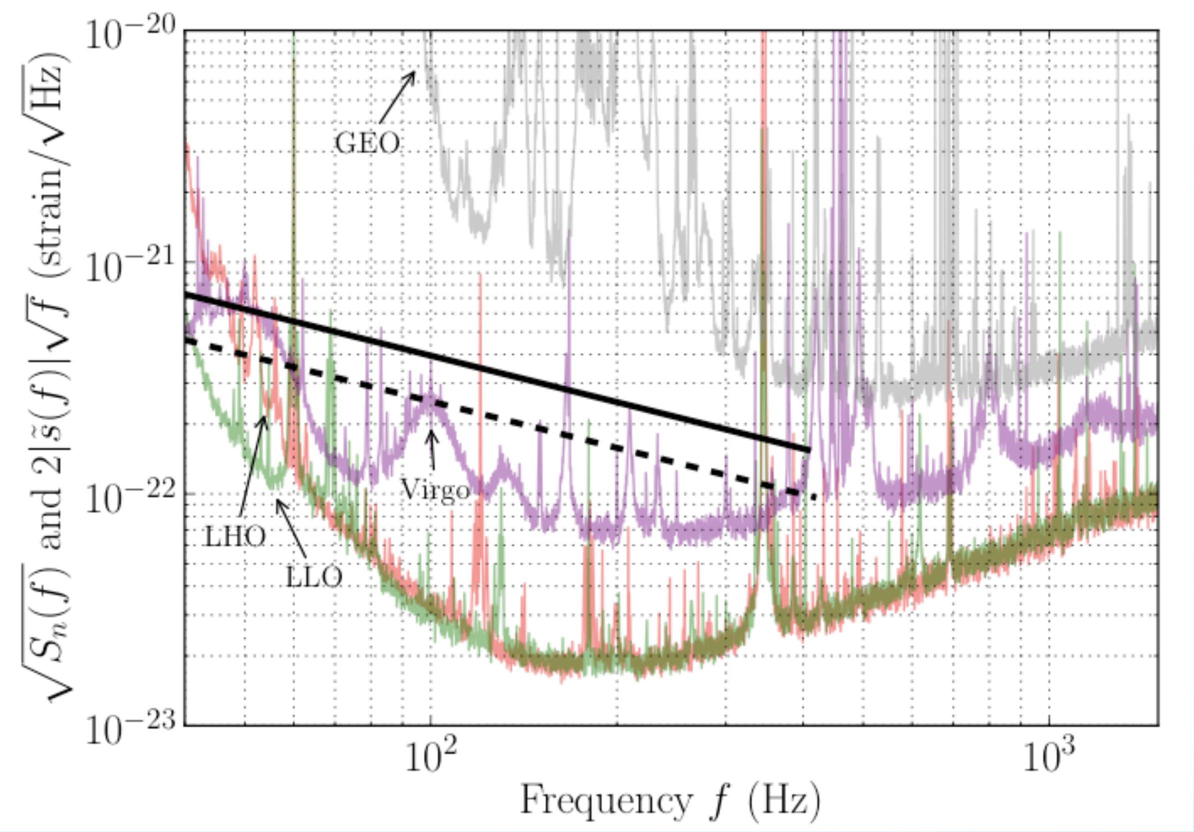
- Beyond lowest order in $|h_{\mu\nu}|$ and v/c
- Dependence on mass ratio & component spins

Frequency dependence

Frequency domain chirp

$|h(f)| \sim f^{-7/6}$

as f increases PN corrections get bigger

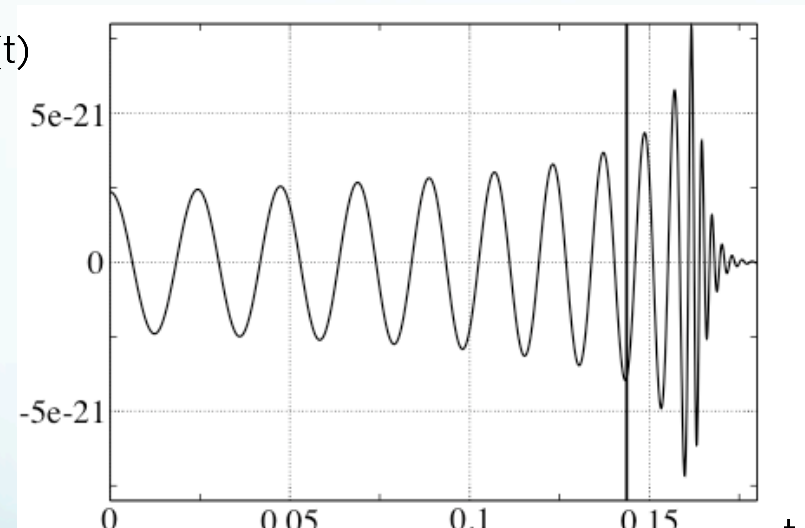


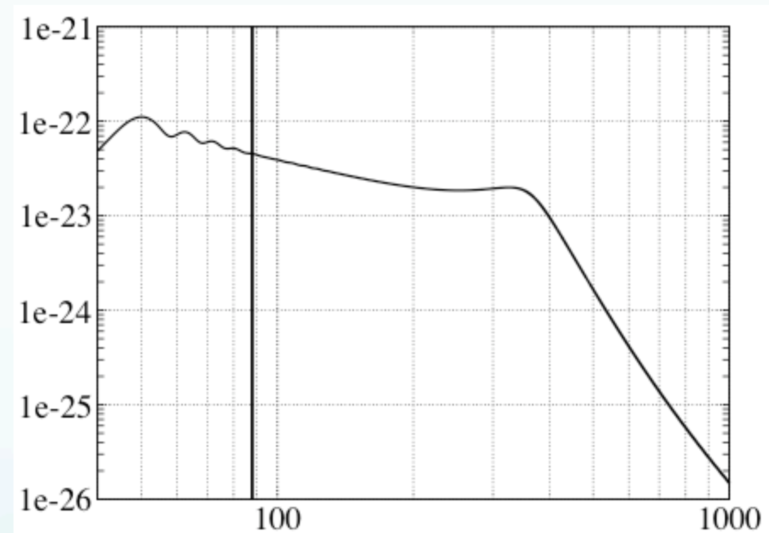
(5,6)$M_\odot$ BBH inspirals vs. detector noises
"Blind hardware injection"

http://www.ligo.org/science/GW100916/

Waveforms with merger/ringdown

- Highly nonlinear & difficult problem
- Combine numerical ('NR') and analytic techniques





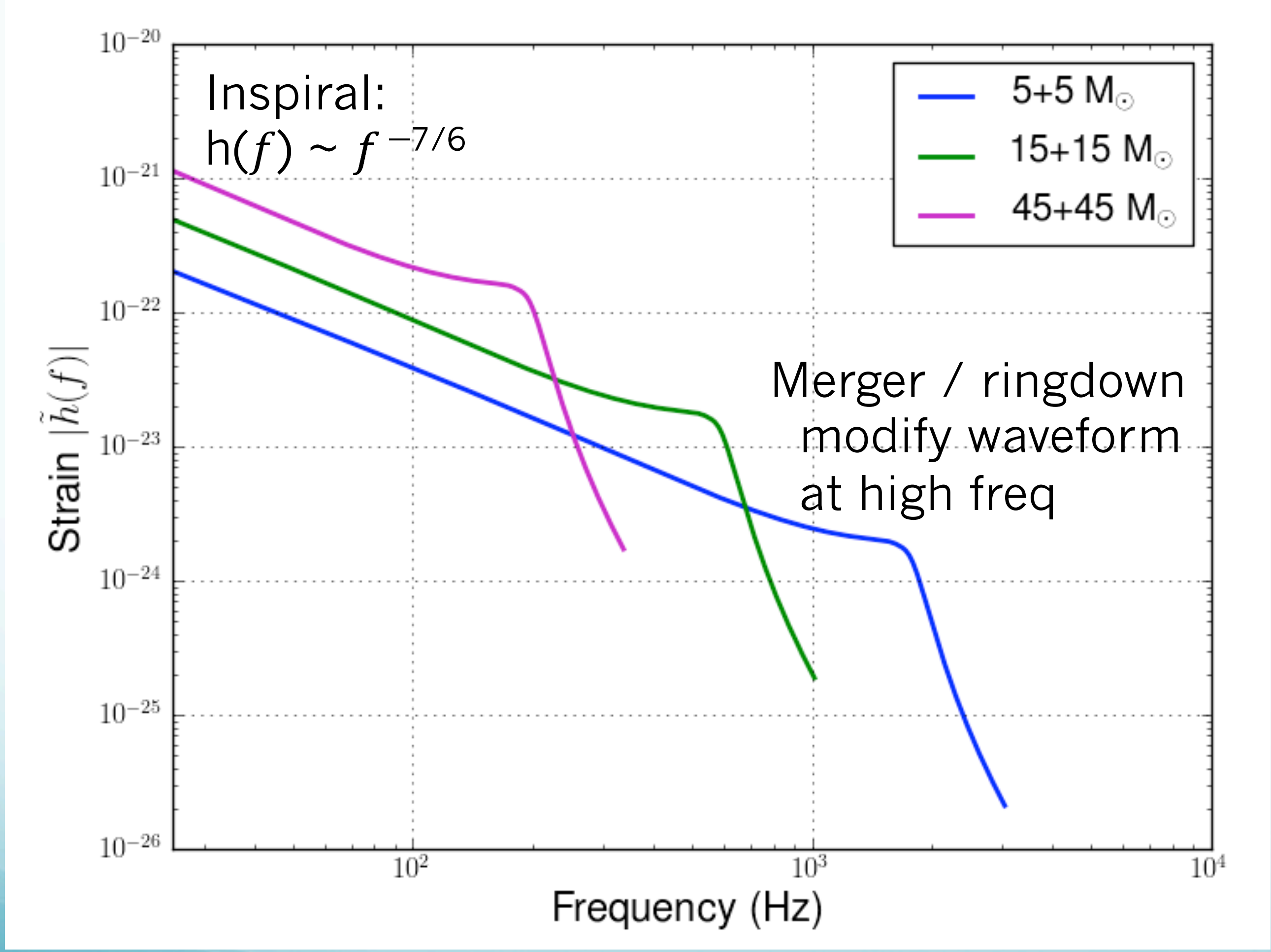
Abadie et al. arXiv:1102.3781

25+25 $M_\odot$ "EOBNR" waveform

Used in search for binaries with black hole(s) : $m_1+m_2 > 4\ M_\odot$

Visualizing an NR solution

Signal in frequency domain



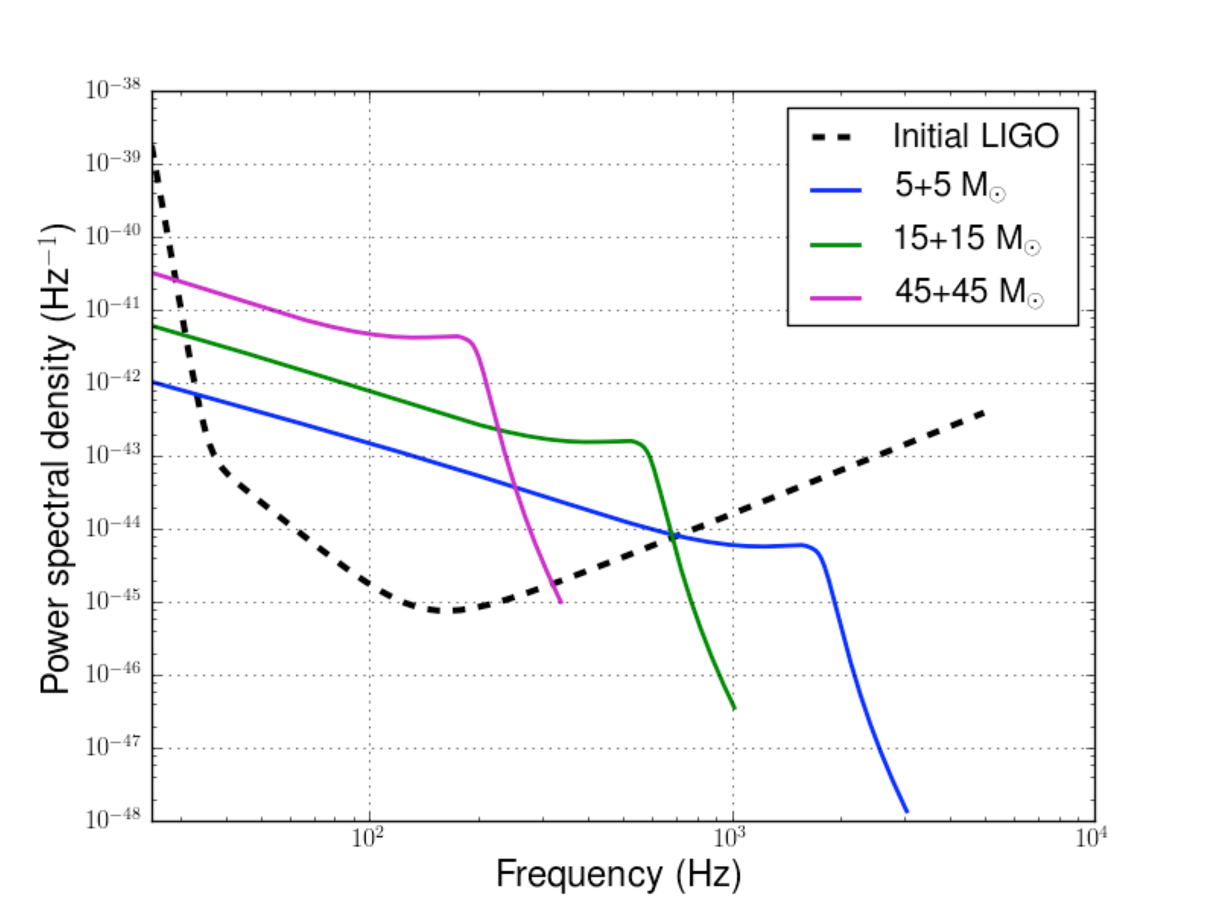
GR has no intrinsic scale
⇒ can freely rescale solutions

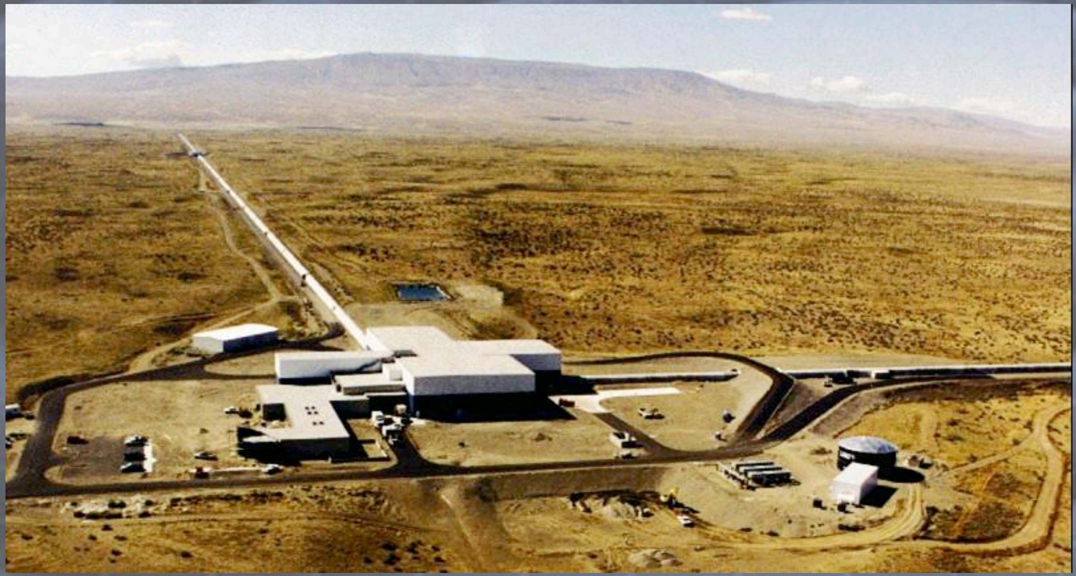
As M increases :

- $|h(f)|$ at fixed distance grows
- maximum GW frequency decreases

Signal vs. noise in freq domain

$|h(f)|^2 \times f$ for optimally aligned & located signals at 30 Mpc

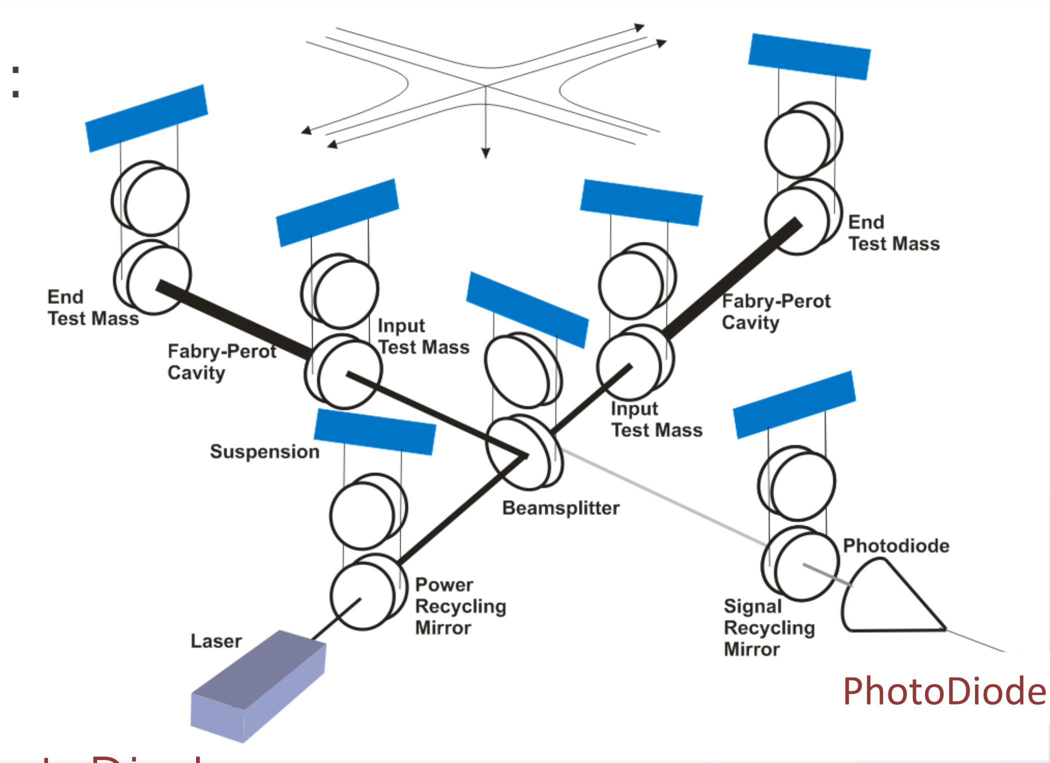




GROUND BASED G.W. DETECTORS

Laser interferometric detection

- Michelson interferometer : end mirrors free to move along arms



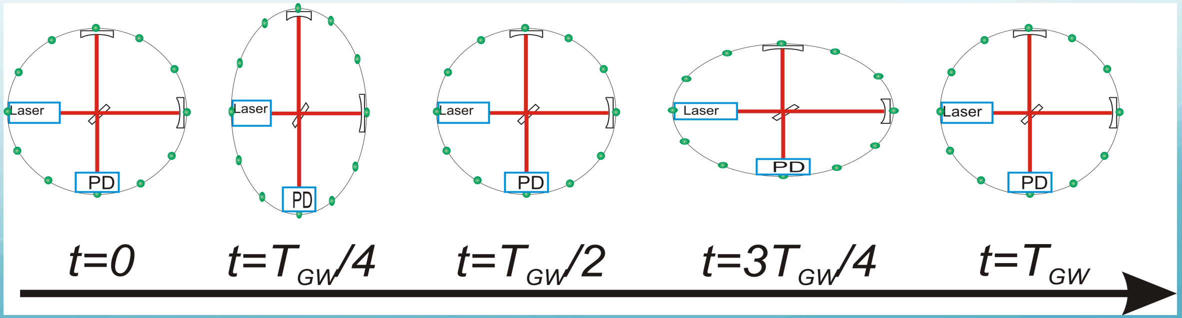
Differential length change

$\delta(L_x - L_y) = h(t) \cdot L$

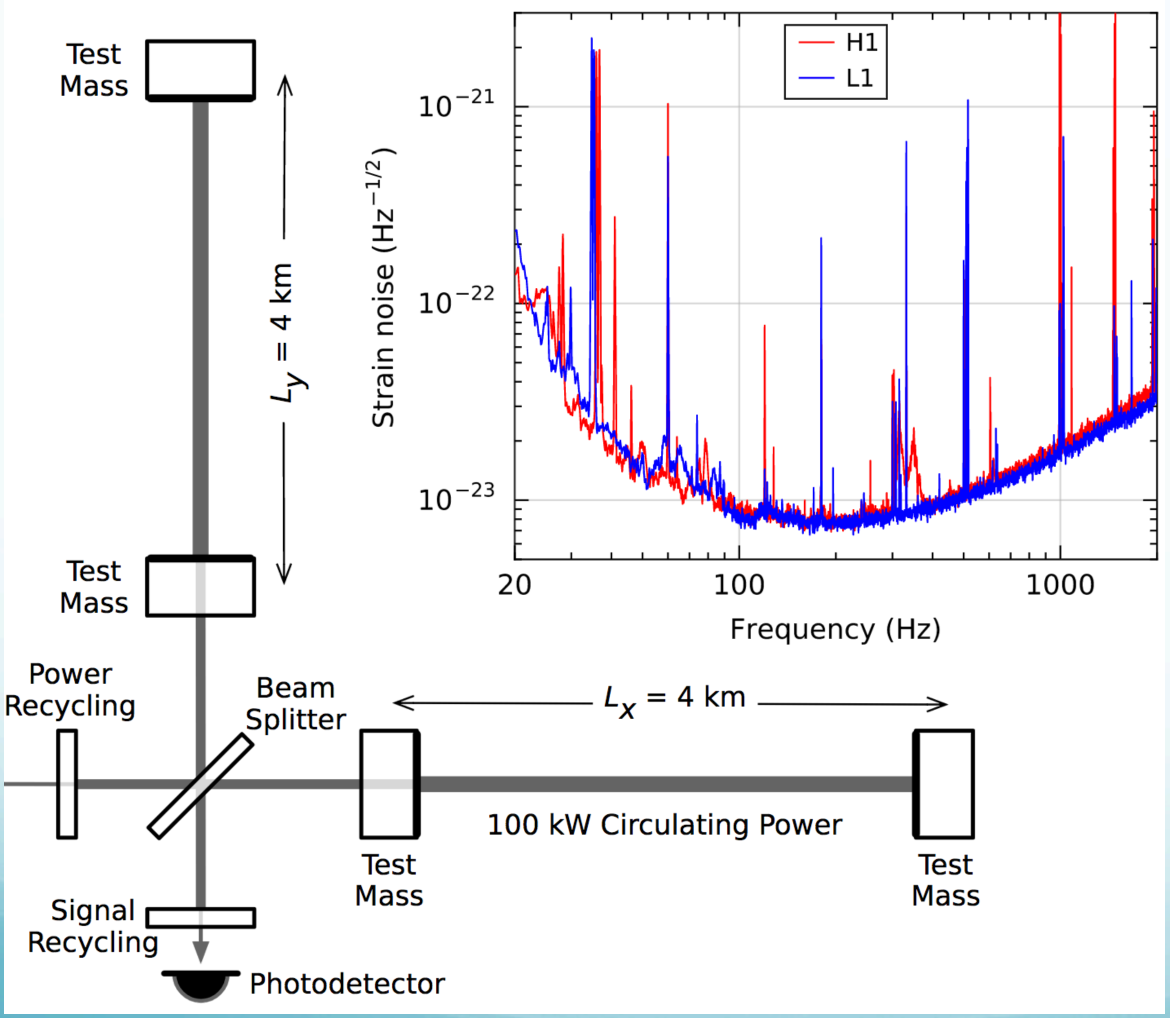
⇒ time of flight difference

⇒ relative phase difference @ beam splitter

⇒ transmitted intensity variation @ PhotoDiode



Getting down to <1e–23

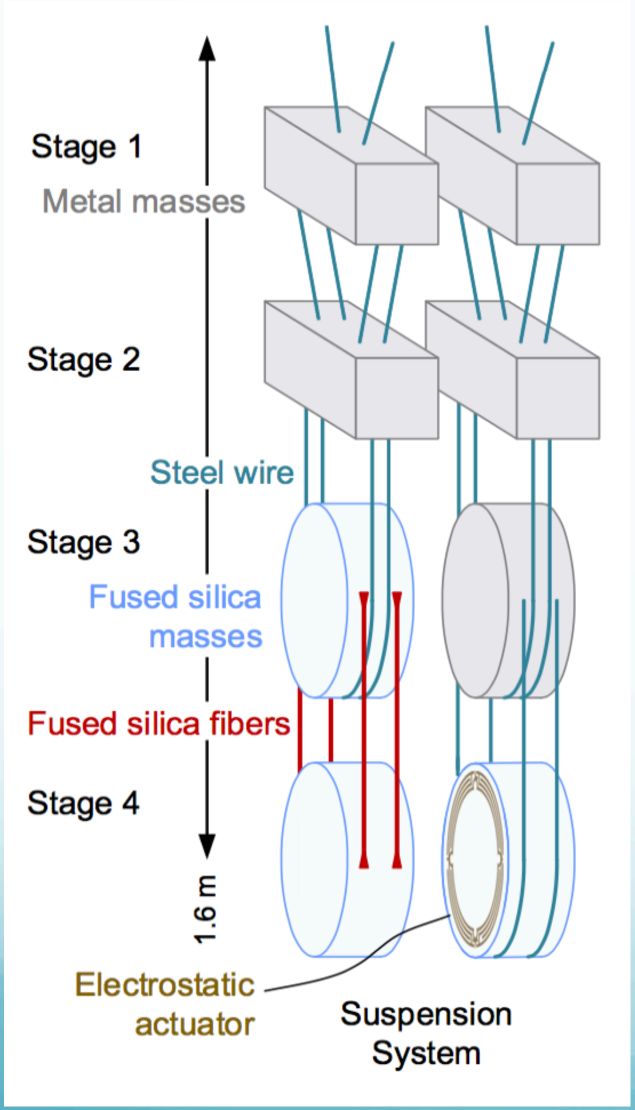


Enhance the signal

- Long arms
- High power ultra-stable laser
- Power recycling (factor ~35)
- Resonant arm cavities (factor ~300)
- Signal recycling

LVC, PRL **116** 061102 (2016)

Getting down to <1e–23



Reduce seismic noise

- Active seismic isolation
- Quadruple pendulum suspension
- ~10 orders of magnitude suppression of displacement noise above 10Hz

Reduce quantum noise

- Inject non-classical 'squeezed' states of photon field

Precision Interferometry = Understanding Measurement Noises

<u>Fundamental Noises</u>

I. Displacement Noises

→ ΔL(f)

- Seismic noise
- Radiation Pressure
- Thermal noise
  - Suspensions
  - Optics

II. Sensing Noises

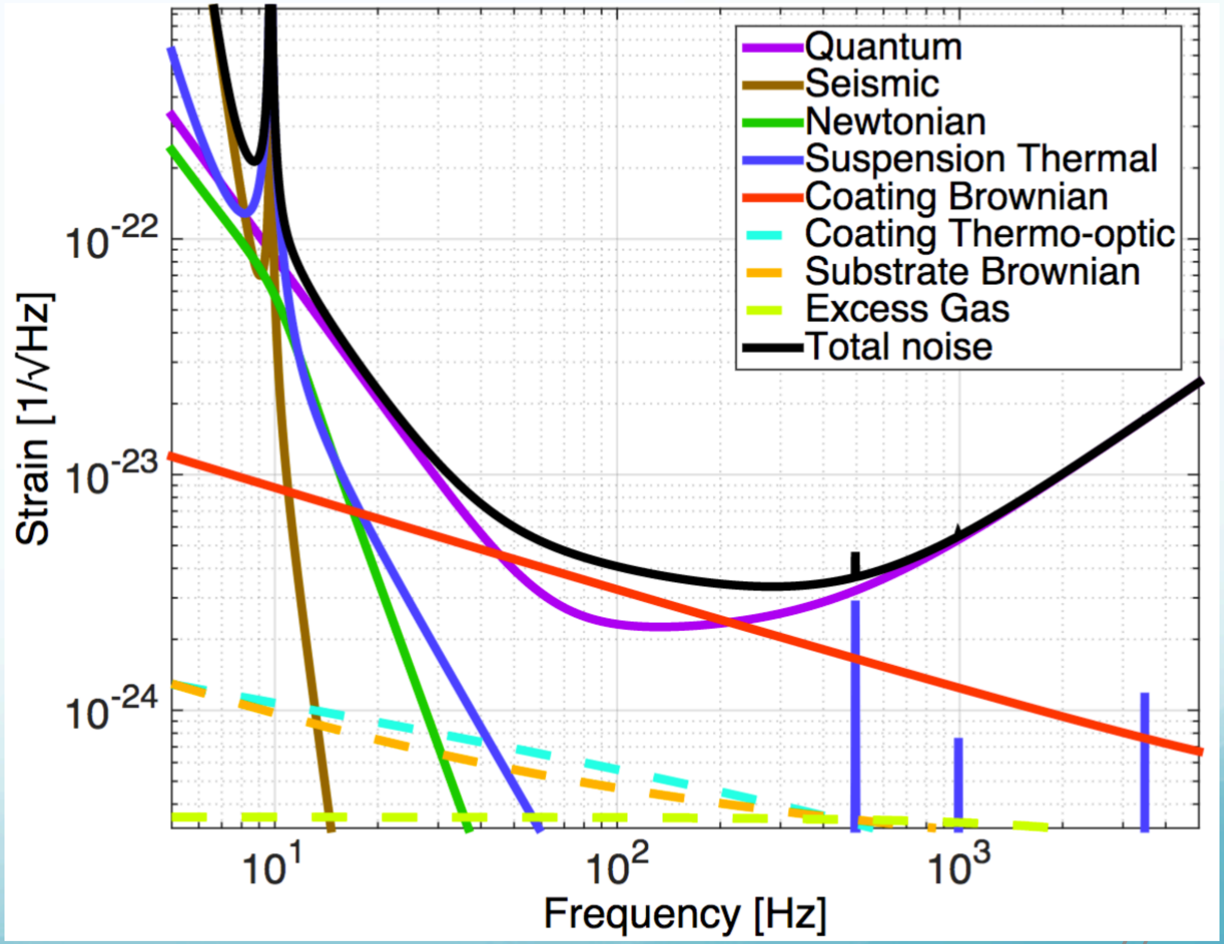
→ $\Delta t_{photon}(f)$

- Shot Noise
- Residual Gas

<u>Technical Noises</u>

→ Hundreds of them…

Advanced LIGO Design Noise Budget



https://dcc.ligo.org/LIGO-T1800044/public

GW signal seen at a detector

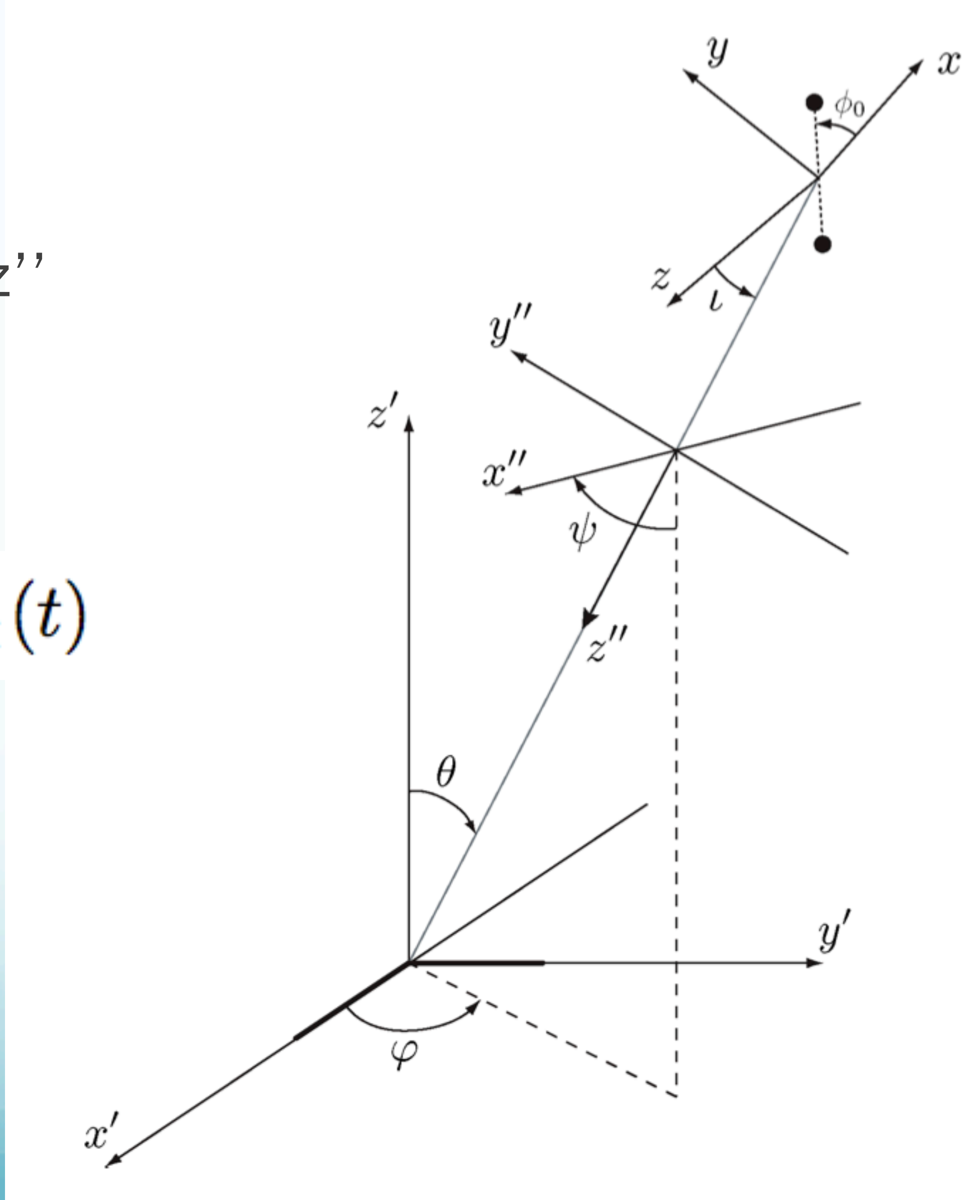
3 Cartesian frames:

- source frame x y z
- radiation frame x'' y'' z''
- detector frame x' y' z'

Strain at the detector:

$$h(t) = F_+ h_+(t) + F_\times h_\times(t)$$

F_+ and $F_\times$: depend on sky position (θ,φ), rotation angle ψ around line of sight



Binary signal seen in 1 detector

- Combine $F_+\cos(\Phi(t))$, $F_\times\sin(\Phi(t))$ components into a single sinusoid

$$h(t) = \frac{A(t)}{\mathcal{D}_{\text{eff}}} \cos\left(\Phi(t) - \theta \right)$$

$$A(t) = -\frac{2G\mu}{c^4} \left[\pi G M f(t)\right]^{\frac{2}{3}}$$

- Effective distance (nb : $\mathcal{D}_{\text{eff}} \geq r$)

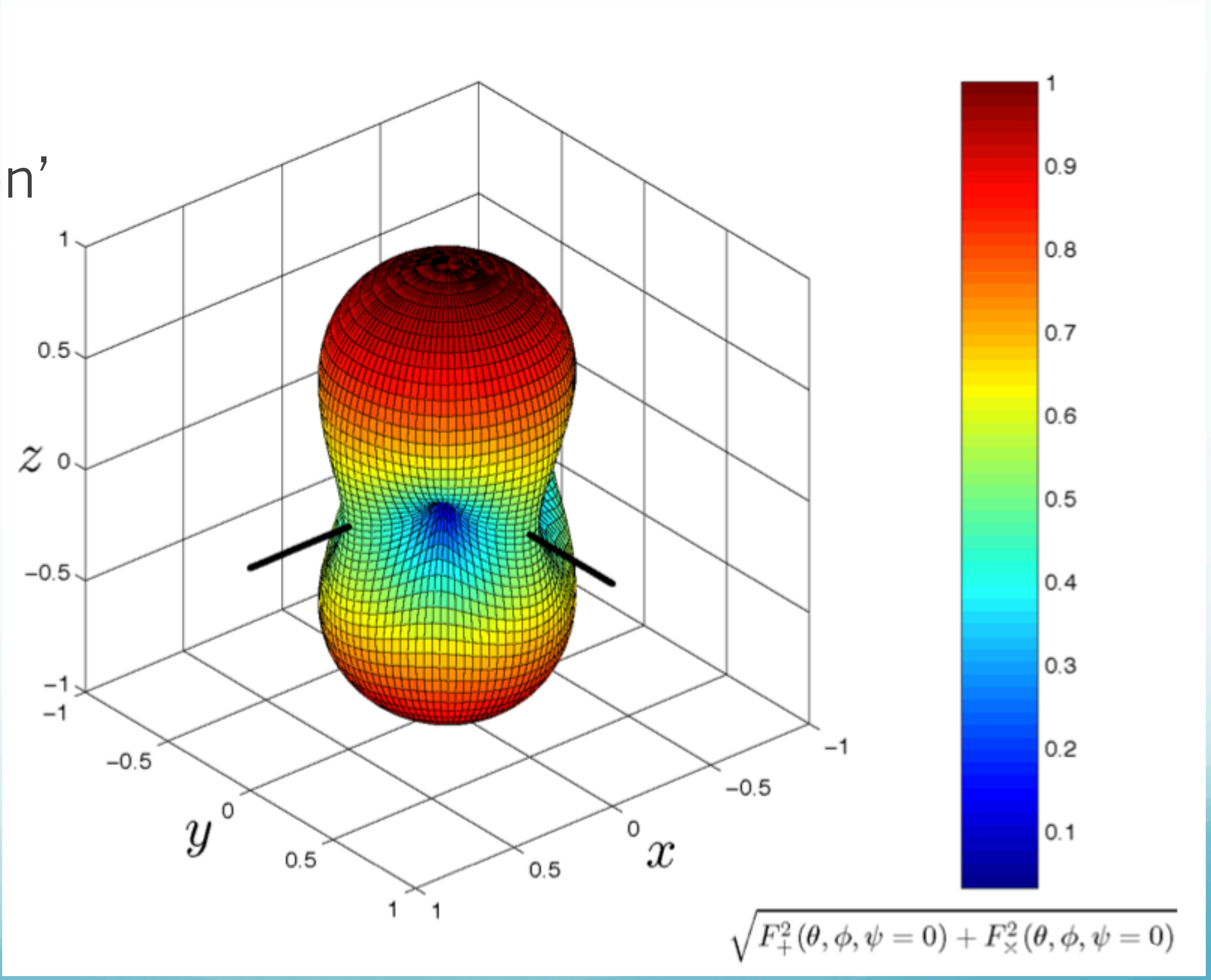
$$\mathcal{D}_{\text{eff}} = \frac{r}{\sqrt{F_+^2 (1+\cos^2\iota)^2/4 + F_\times^2 \cos^2\iota}}$$

- Phase shift

$$\tan\theta = \frac{F_\times 2\cos\iota}{F_+(1+\cos^2\iota)}$$

Detector response to inspiral signal

- take $\iota = 0$
  i.e. 'face on' binary



The detection problem

The statistical problem

- CBC signals arrive at the detector all the time!
- The great majority are ‘too weak to detect’
  - Sources are not within sensitive volume of detector
  - Cannot extract useful (astrophysical) info
- Detector output is signal plus noise:

$$\mathbf{s(t) = h(t) + n(t)}$$

- Detection means:
  The data favour nonzero signal relative to no signal

⇒ tell the difference between

$$s(t) = h(t) + n(t) \qquad \text{vs.} \qquad s(t) = 0 + n(t)$$

Signal and noise hypotheses

- Hypothesis $\mathbf{H_1}$: $s(t) = s_1(t) = h(t) + n(t)$ $h(t) \neq 0$
- Hypothesis $\mathbf{H_0}$: $s(t) = s_0(t) = n(t)$
- **Bayes' rule:**

$$\frac{P(\mathbf{H}_1|d)}{P(\mathbf{H}_0|d)} = \frac{P(d|\mathbf{H}_1)}{P(d|\mathbf{H}_0)} \times \frac{P_i(\mathbf{H}_1)}{P_i(\mathbf{H}_0)}$$

d, "data" → s(t)

Posterior Odds Ratio — Likelihood Ratio ('Bayes Factor') — Prior Odds Ratio

- Prior odds depends on astrophysical coalescence rate (mergers /volume /time) – highly uncertain!

Neyman-Pearson optimal statistic

- Λ(d) is optimal if it **maximizes detection probability at a fixed value of false alarm probability**
- Can be proved that **likelihood ratio**

$$\Lambda(\mathrm{d}) = \Lambda_{\mathrm{opt}} = \frac{P(d|\mathbf{H}_1)}{P(d|\mathbf{H}_0)}$$

  is an optimal statistic for a known signal h(t)
- Any *monotonic increasing function* of Λ_{opt} gives same ranking of possible data *d* – also optimal

Statistics of (Gaussian) noise

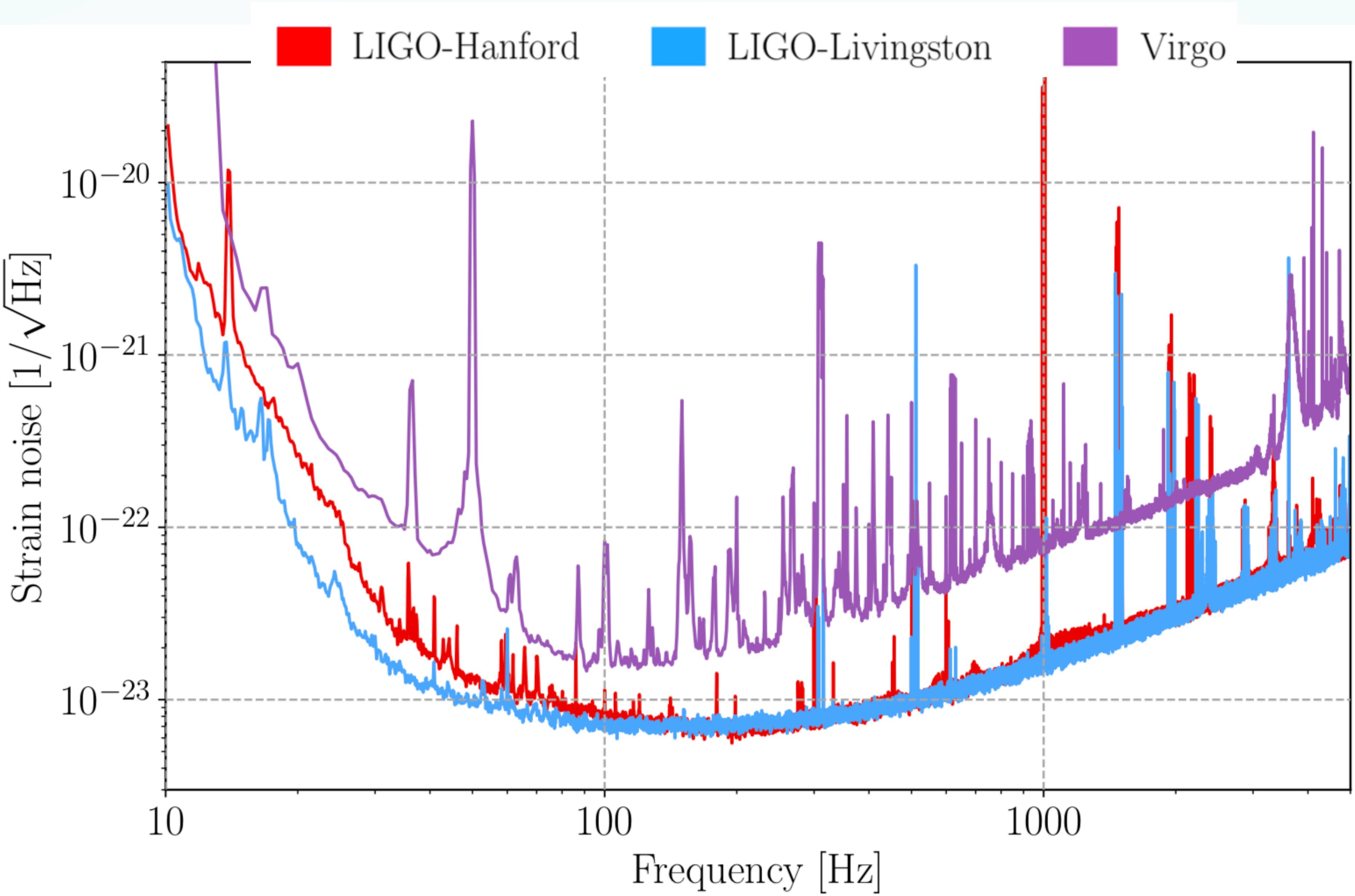
To calculate likelihood, need statistics of noise P(n(t))

- Simplest assumption : stationary process, describe in the frequency (Fourier) domain n(f)
- Autocorrelation function $R(\tau) = \langle n(t+\tau)\, n(t) \rangle$
- F.T. ⇒ **PowerSpectralDensity** $S_n(f)$

$$\langle n^*(f) n(f') \rangle = \delta(f - f') \frac{1}{2} S_n(f)$$

Noise at different frequencies not correlated

- Quantity linear in GW strain : amplitude spectral density 'ASD' $\sqrt{S_n(f)}$ units : $\mathtt{strain}/\mathrm{Hz}^{1/2}$


LIGO-Hanford
LIGO-Livingston
Virgo
10^{-20}
10^{-21}
10^{-22}
10^{-23}
Strain noise $[1/\sqrt{\mathrm{Hz}}]$
10
100
1000
Frequency [Hz]

Likelihood for noise vs. signal

- Noise likelihood : under $\mathbf{H_0}$, n(f) = s(f)

$$P(s(f)|\mathbf{H}_0) = \mathcal{N} \exp\left\{-\frac{1}{2}\int_{-\infty}^{\infty} df \, \frac{|s(f)|^2}{\frac{1}{2}S_n(f)}\right\}$$

- Signal likelihood : under $\mathbf{H_1}$, n(f) = s(f) – h(f)

$$P(s(f)|\mathbf{H}_1) = \mathcal{N} \exp\left\{-\frac{1}{2}\int_{-\infty}^{\infty} df \, \frac{|s(f) - h(f)|^2}{\frac{1}{2}S_n(f)}\right\}$$

Scalar products and likelihood ratio

- Define **scalar product** of data streams a(t), b(t)

$$\langle a|b\rangle = \mathrm{Re}\int_{-\infty}^{\infty} df\, \frac{a^*(f)b(f)}{\frac{1}{2}S_n(f)}$$

- Usual properties: $\langle a|b\rangle = \langle b|a\rangle$, $\langle a|a\rangle \geq 0$ etc.
- Rewrite likelihoods :

$$P(d|\mathbf{H}_0) = \mathcal{N}e^{-\frac{1}{2}\langle s|s\rangle}$$

$$P(d|\mathbf{H}_1) = \mathcal{N}e^{-\frac{1}{2}\langle s-h|s-h\rangle} = \mathcal{N}e^{-\frac{1}{2}\langle s|s\rangle+\langle s|h\rangle-\frac{1}{2}\langle h|h\rangle}$$

- Likelihood ratio $\Lambda_{\mathrm{opt}} = \dfrac{P(d|\mathbf{H}_1)}{P(d|\mathbf{H}_0)} = e^{\langle s|h\rangle-\frac{1}{2}\langle h|h\rangle}$

Optimal matched filter

- $\langle h|h \rangle$ is constant for a fixed signal, e^x is monotonic
- Therefore we can also use $\langle s|h \rangle$ as our statistic

  Known as 'matched filter'

  Linear in the detector output s

$$\langle s|h\rangle = \mathrm{Re}\int_{-\infty}^{\infty} df\, K^*(f)s(f), \quad K(f) = \frac{h(f)}{\frac{1}{2}S_n(f)}$$

- Expected value of $\langle s|h \rangle$ under $\mathbf{H_0}$ is = 0
- Expected value of $\langle s|h \rangle$ under $\mathbf{H_1}$ is = $\langle h|h \rangle$
- Variance of $\langle s|h \rangle$ is $\sigma^2 = \langle h|h \rangle$

Signal-to-noise ratio (SNR)

- Rescale the matched filter : $\rho = \frac{\langle s|h\rangle}{\sqrt{\langle h|h\rangle}}$
- Variance $\sigma^2(\rho) = 1$
- Mean $\bar{\rho}_{;0} = 0$ (noise)

  $\bar{\rho}_{;1} = \sqrt{\langle h|h\rangle}$ (signal)
- $\bar{\rho}$ is "expected" / "optimal SNR" of signal h(t)
- Distribution of ρ :

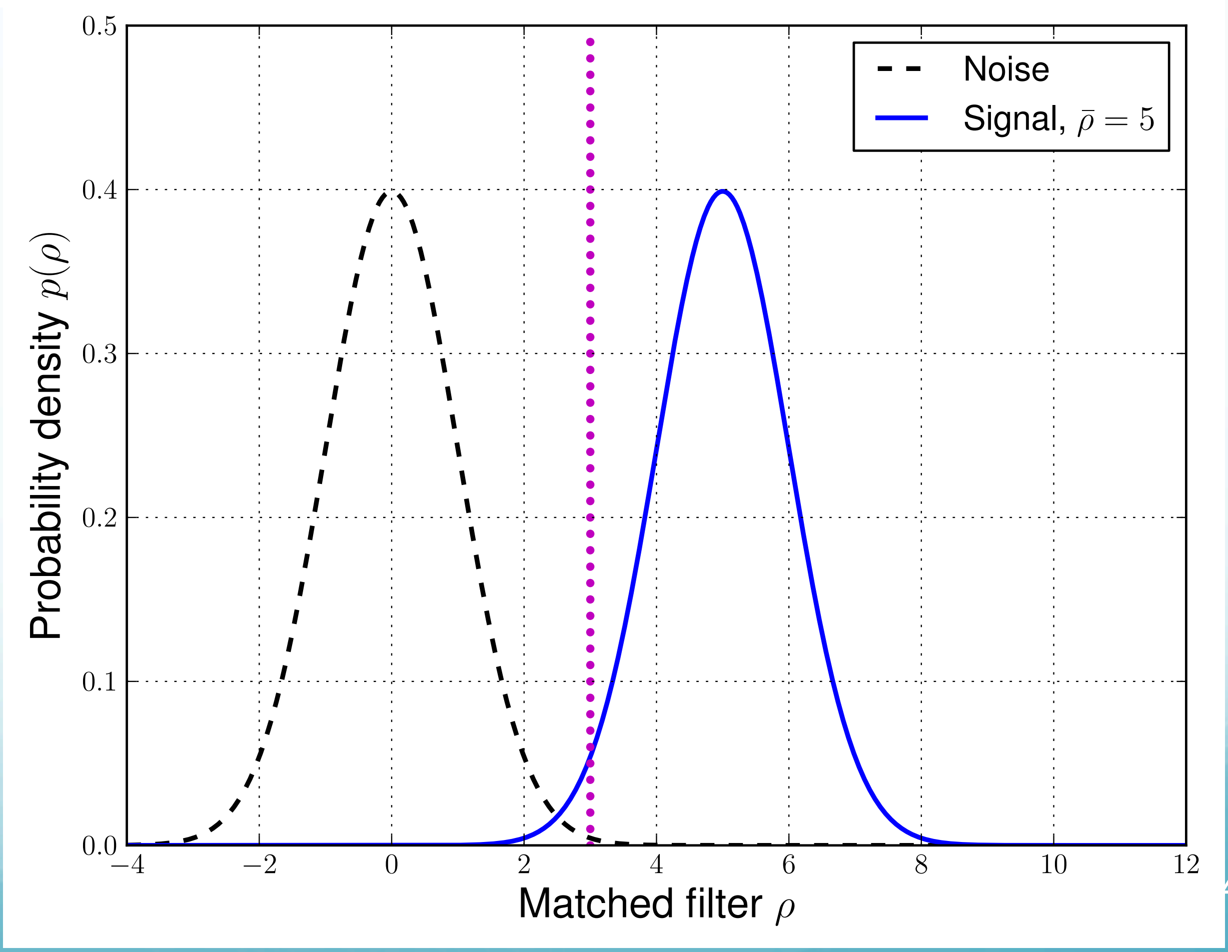
$$p(\rho|\bar{\rho})\, d\rho = \frac{1}{\sqrt{2\pi}} e^{-(\rho-\bar{\rho})^2/2}\, d\rho$$

Matched filter output statistics

$$p(\rho|\bar{\rho})\,d\rho = \frac{1}{\sqrt{2\pi}} e^{-(\rho-\bar{\rho})^2/2}\,d\rho$$

Expected value in presence of signal *h*

$$\bar{\rho} = \sqrt{\langle h|h\rangle}$$



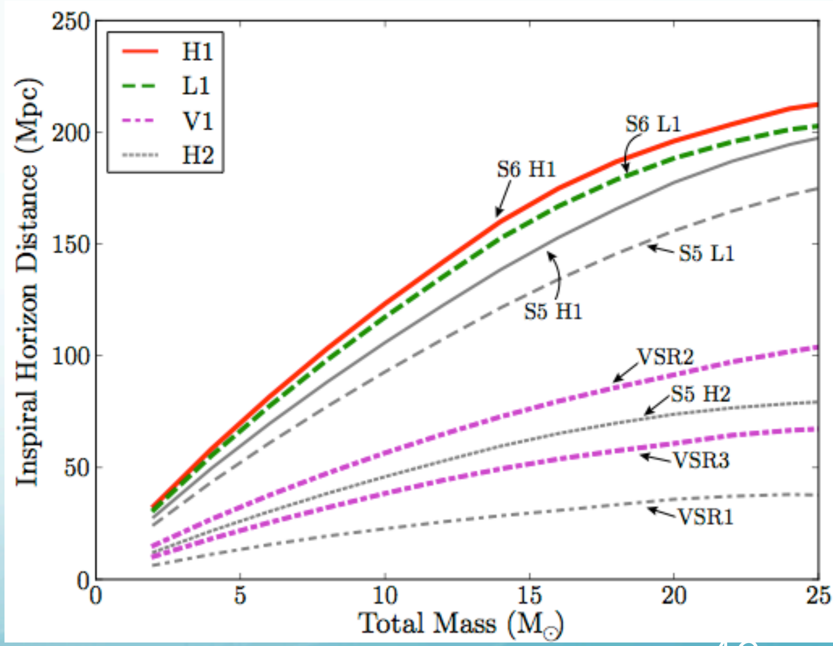
Horizon distance

- Farthest distance D_h where a merger could produce a given expected SNR $\overline{\rho}$, e.g. = 8

$$h(f) = \frac{1\,\mathrm{Mpc}}{D_{\mathrm{eff}}} \mathcal{A}_{1\mathrm{Mpc}}\, f^{-7/6} \exp(i\Psi(f; \mathcal{M}, M))$$

$$\left(\frac{S}{N}\right)^2 = 4 \int_0^\infty \mathrm{d}f\, \frac{|\tilde{h}(f)|^2}{S_n(f)}$$

D_h depends on binary masses & detector noise spectrum



J. Abadie *et al.*, arXiv:1111.7314

Signal parameters seen in h(t)

- Signal h(t) is not unique (not a ‘simple hypothesis’)
- Described by parameters “θ”
  - Amplitude $\propto A_{1\mathrm{Mpc}}/D_{\mathrm{eff}}$
    Effective distance D_{eff} encodes physical distance D and geometry relative to the detector
  - Coalescence phase ϕ_0
  - Coalescence time t_0
  - Masses m_1, m_2, component spins, ...
- Theoretically correct treatment ; evaluate likelihood $p(d|\mathbf{H_1}(\theta))$ for all θ, marginalize (integrate) over θ

CBC signal parameters I

- Amplitude: Easy, the matched filter doesn't care about amplitude of h

$$\rho = \frac{\langle s|h\rangle}{\sqrt{\langle h|h\rangle}}$$

  - The value of ρ is a *measurement* of expected SNR $\overline{\rho}$
  - Proportional to $A_{1\mathrm{Mpc}}/D_{\mathrm{eff}}$ for a signal

CBC signal parameters II

- Coalescence phase: Easy, use ‘cos’ and ‘sin’ filters

$$\Psi(f) = -2\phi_0 + \Psi'(f)$$

$$\langle s|f^{-7/6}e^{i\Psi(f)}\rangle = \cos 2\phi_0 \langle s|f^{-7/6}e^{i\Psi'(f)}\rangle + \sin 2\phi_0 \langle s|f^{-7/6}(-i)e^{i\Psi'(f)}\rangle$$

$$= \cos 2\phi_0 \cdot x + \sin 2\phi_0 \cdot y$$

  - Can show that $|z| = |x + iy| = \sqrt{x^2 + y^2}$ is an optimal statistic if the phase ϕ_0 is not known.
  - z is a *complex matched filter* :

$$z = \frac{2A}{D_{\text{eff}}} \int_{f_{\min}}^{f_{\max}} df \, \frac{\tilde{s}(f) f^{-7/6} e^{-i\Psi'(f)}}{\frac{1}{2} S_n(f)}$$

CBC signal parameters III

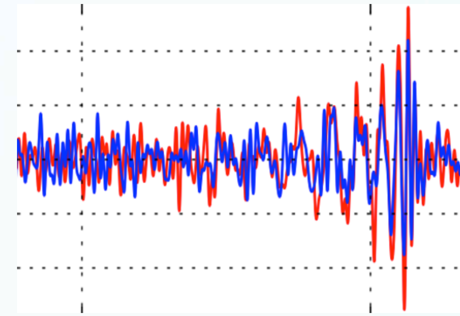
- Coalescence time: Easy.
  - Rewrite $\Psi'(t_0) = \Psi'(t_0 = 0) \cdot e^{2\pi i f t_0}$
  - Get a matched filter *time series* :

$$z(t_0) = \frac{2A}{D_{\text{eff}}} \int_{f_{\min}}^{f_{\max}} df \, \frac{s(f) f^{-7/6} e^{-i\Psi'(f;t_0=0)}}{\frac{1}{2} S_n(f)} e^{-2\pi i f t_0}$$

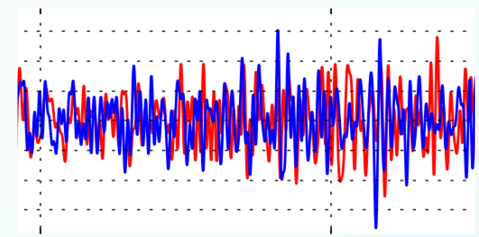
  - It's just a Fourier transform ! Can use FFTs etc.

Modelled binary merger search

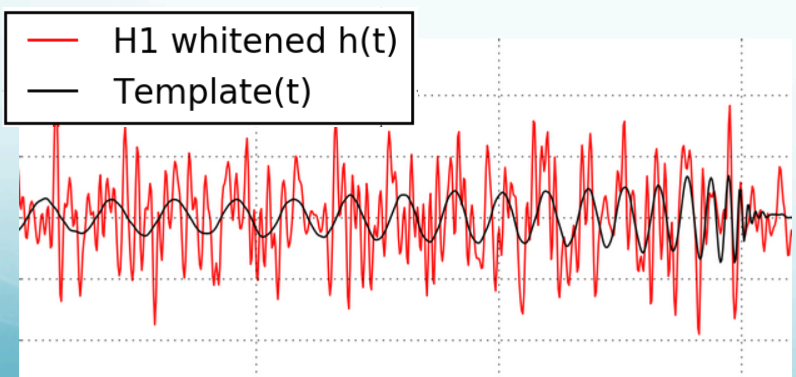
- GW150914 'easily' visible in (minimally filtered) detector output



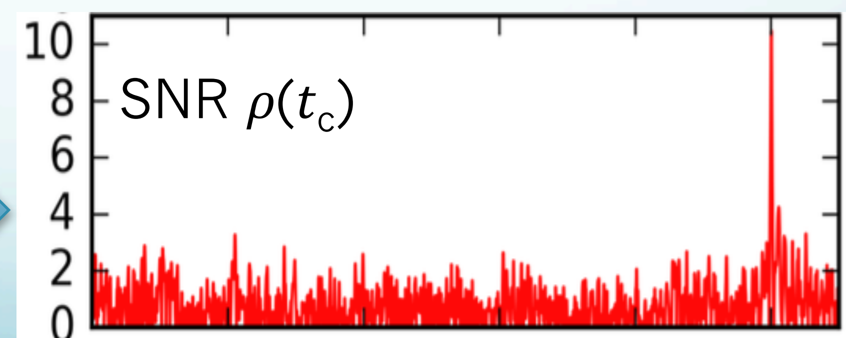
- Most events in O1/O2 were not



- eg GW151226 detected *only* by matched filtering



$\Sigma\, \mathrm{d}f$



time t

merger time t_c

Signal geometry and template banks

How many filters do we need?

- Different masses $\theta = \{m_1, m_2\}$ require different filters
- If there is a signal with parameters θ and we use filter parameters $\theta' \neq \theta$ we do not have an optimal search
  - Given a fixed SNR ρ^* for detection, the probability that the signal exceeds ρ^* will be smaller for a mismatched template
  - How much 'lack of match' is acceptable?
- Define 'match' $M \leq 1$

$$M = \bar{\rho}/\bar{\rho}_{\mathrm{opt}}$$
$$= (\text{SNR for template } \theta')/(\text{SNR for optimal template } \theta)$$

Loss in search sensitive volume

- Assume binary mergers are uniform in space
- Volume of space where signals can be detected with $\bar{\rho} > \rho^*$ is $\propto D_{max}(\rho^*)^3$
- Optimal template: $$\bar{\rho}_{\rm opt} \propto \frac{\rm A}{D_{\rm max}}$$
- Non-optimal template: $$\bar{\rho} \propto M \frac{\rm A}{D_{\rm max}}, \quad M \leq 1$$
- Thus $D_{max}(\rho^*) \propto M/\rho^*$, sensitive volume $\propto (M/\rho^*)^3$

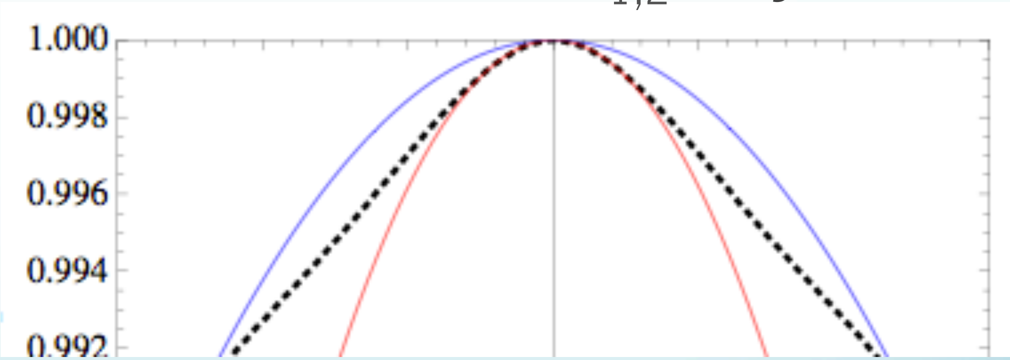
(Mis)match of templates

- Use normalized templates $h(\theta, t_0, \phi_0)$: $\langle h|h \rangle = 1$
- Match *M* for small mass differences :

$$M(\theta, \Delta\theta) = \max_{t_0, \phi_0} \langle h(\theta) | h(\theta + \delta\theta) \rangle$$

max over t_0, ϕ_0 ensures differences due to $m_{1,2}$ only

- Expand near local maximum at $\Delta\theta = 0$:



$$M(\theta, \Delta\theta) = 1 + \frac{1}{2} \frac{\partial^2 M}{\partial\theta_i \partial\theta_j} \Delta\theta_i \Delta\theta_j + \mathcal{O}(\Delta\theta_i^3)$$

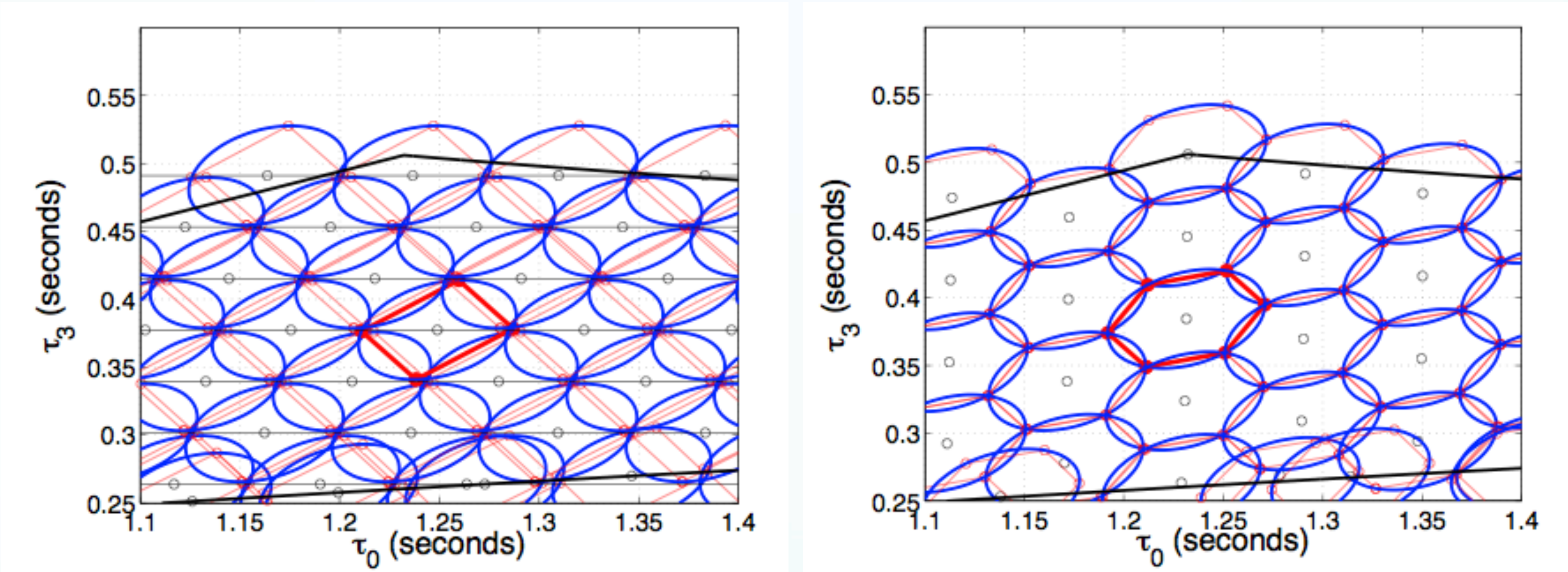
Mismatch metric

- Local deviation from $M = 1$ defines a **metric** over θ_i

$$1 - M = \text{``}ds^2\text{''} = g_{ij}\Delta\theta_i\Delta\theta_j, \quad g_{ij}(\theta) = -\frac{1}{2}\frac{\partial^2 M}{\partial\theta_i\partial\theta_j}$$

- Calculate M(θ,$\Delta\theta$) explicitly → find g_{ij}
- Sometimes may find coordinates where g_{ij} is (nearly) constant
- Use a **regular lattice** of templates
  - Ensures that no point in space is further than some maximum distance from a template
  - ds^2_{max} : “maximal mismatch”

Template bank placement

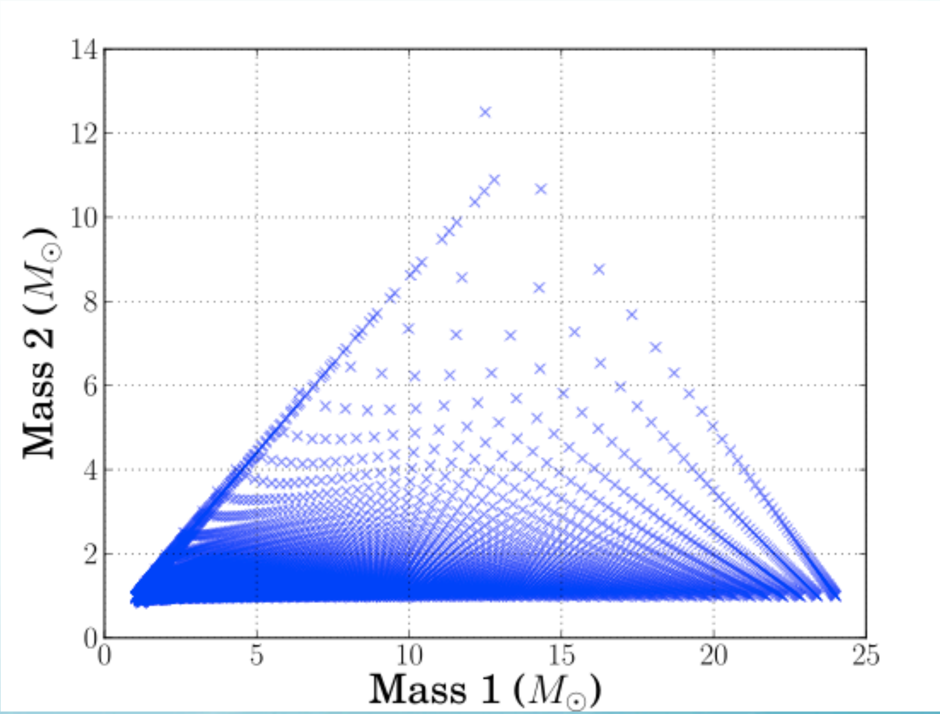


- Hexagonal bank is more efficient at covering space
- "Chirp time" coordinates τ_0, τ_3 : functions of $m_{1,2}$

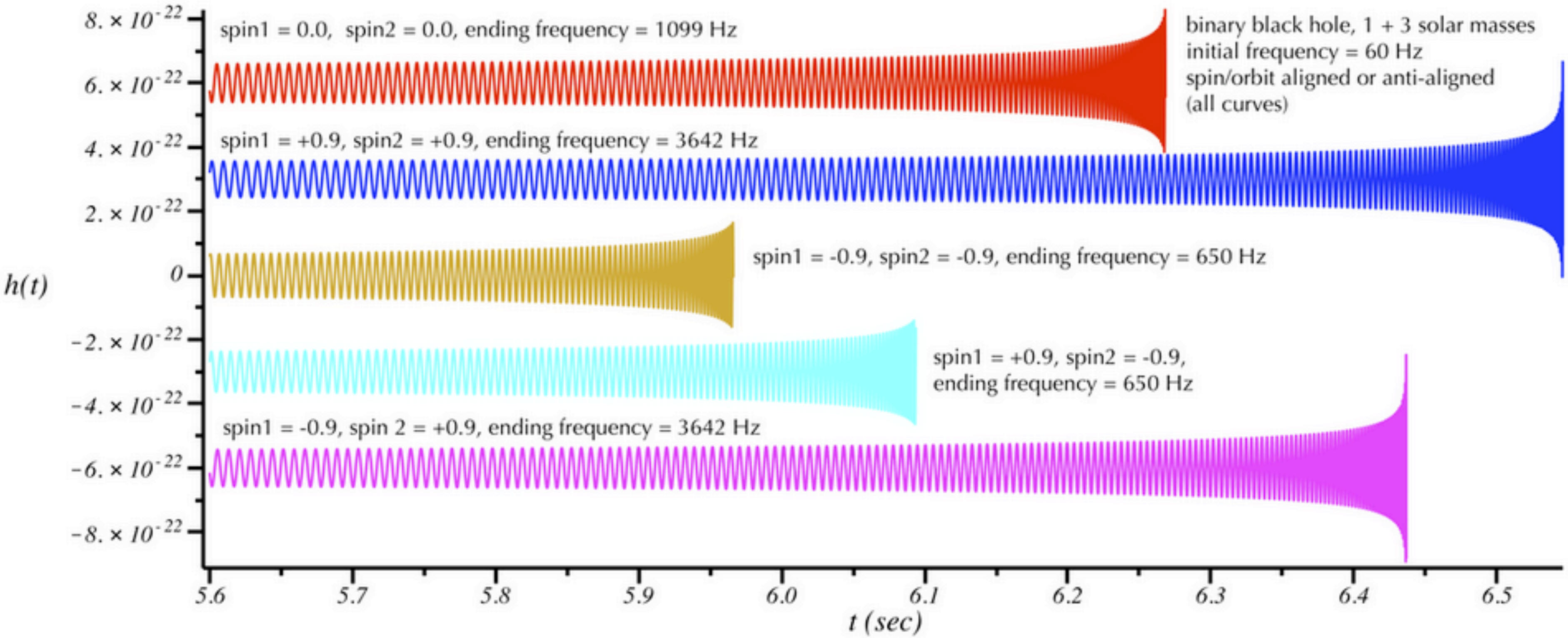
T. Cokelaer, arXiv:0706.4437

'Geometric' template bank

- Minimal match 0.97 (maximal mismatch 0.03)
  - ~10% maximum possible loss of sensitive volume
- Component masses $1 < m_{1,2}/M_\odot < 24$
- Max mtotal = 25 $M_\odot$
- Order 10,000 templates
- Computationally feasible to search ✓



Effects of spin on BBH signals

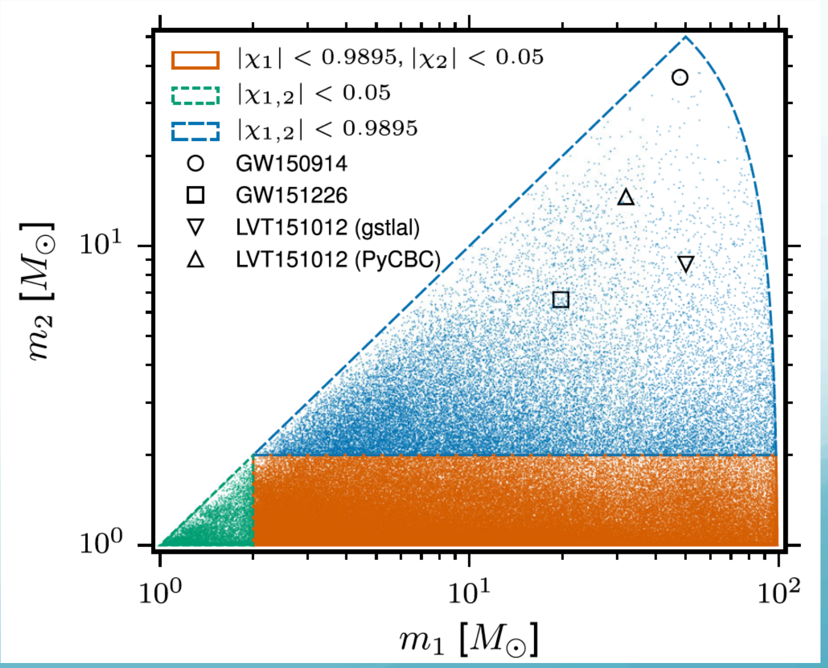


- last stages of inspiral/merger last for more/fewer cycles, end at higher/lower frequency

https://www.soundsofspacetime.org/spinning-binaries.html

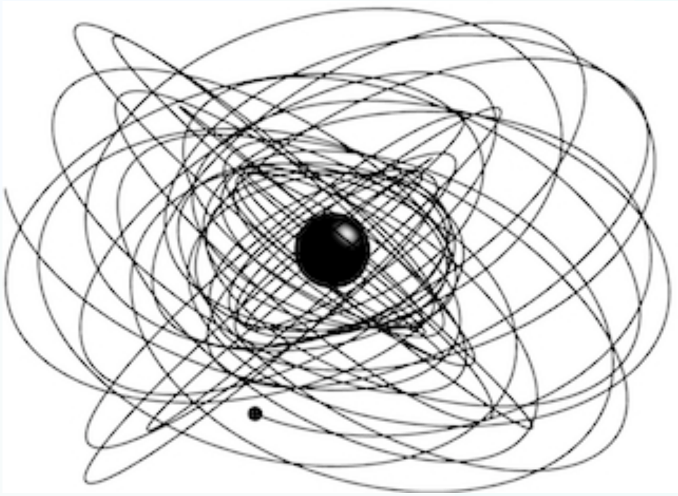
Stochastic / aligned spin banks

- With nonzero spins s_1, s_2 parameter space becomes multidimensional (eg 4d for 'aligned' spin)
- Metric far from ~constant for IMR templates
- General method : 'stochastic' placement
  - Pseudorandom choice of test points
  - Reject if 'too close' to already accepted point
  - e.g. LIGO O1 bank



Challenges

- Signals may be complicated / uncertain / unpredictable
  - many free parameters
  - GR is hard theory to calculate

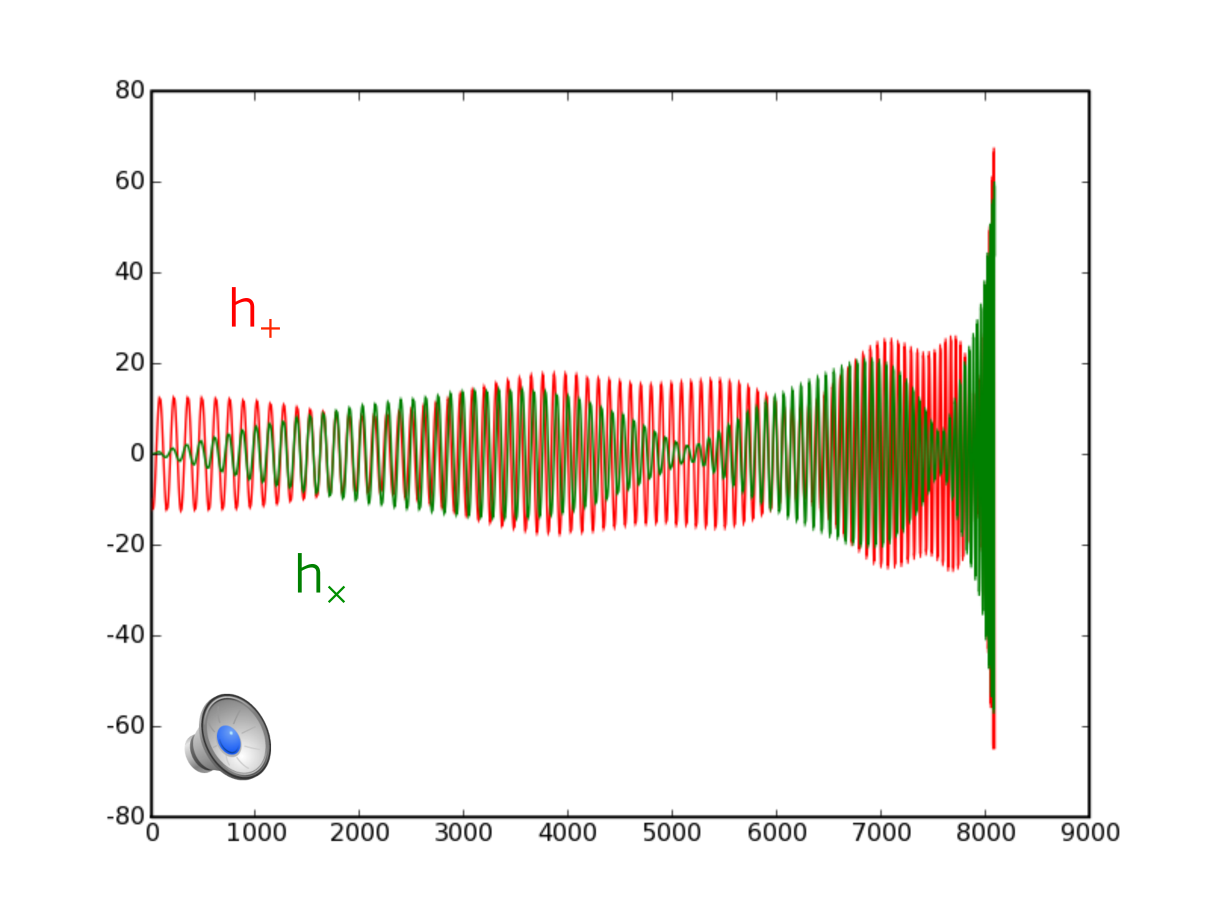


EMRI orbit (S. Drasco)

- **Noise may be complicated**
  - non-Gaussian – i.e. containing loud non-GW events 'glitches'
- **Noise may be unpredictable**
  - Some types of 'glitch' not (so far) diagnosed or removed

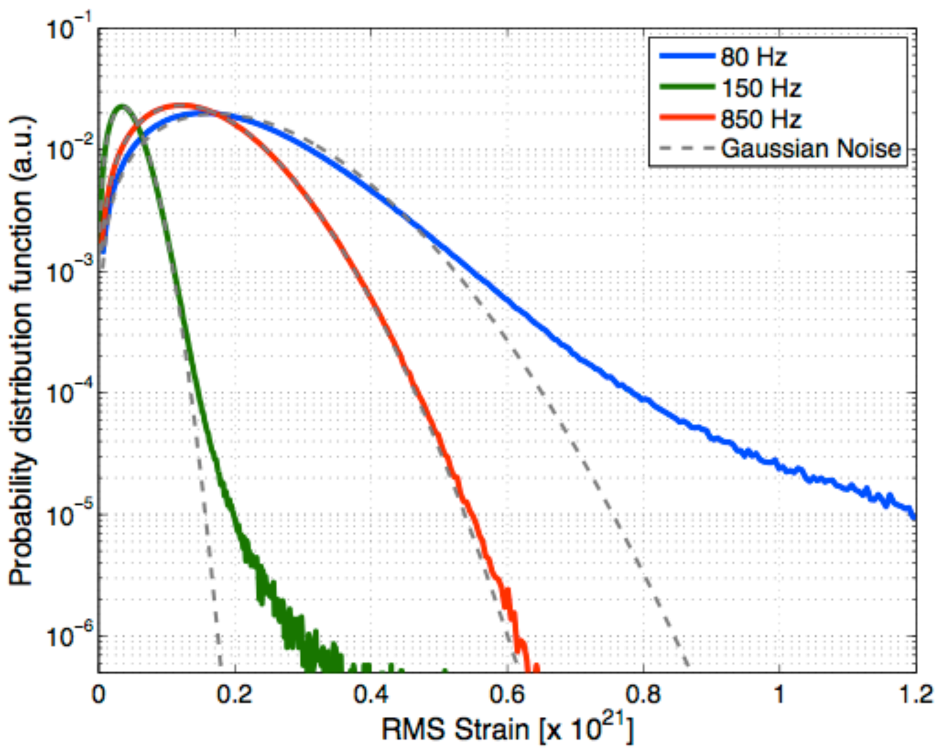
Also, computing storage and power are not infinite !

A spinning precessing waveform



credit: A. Lundgren

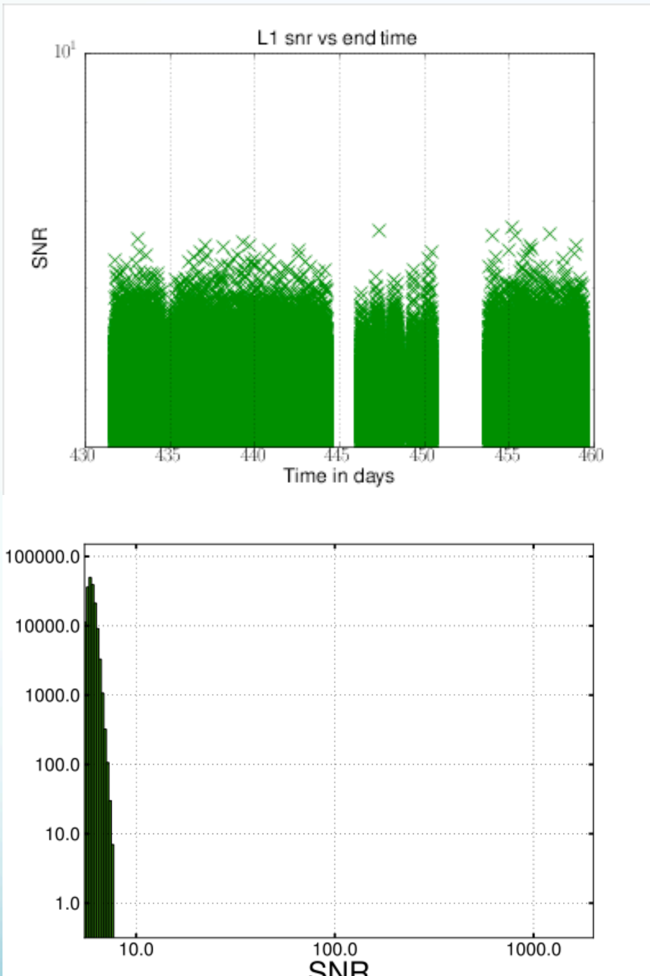
Real detector noise is not Gaussian

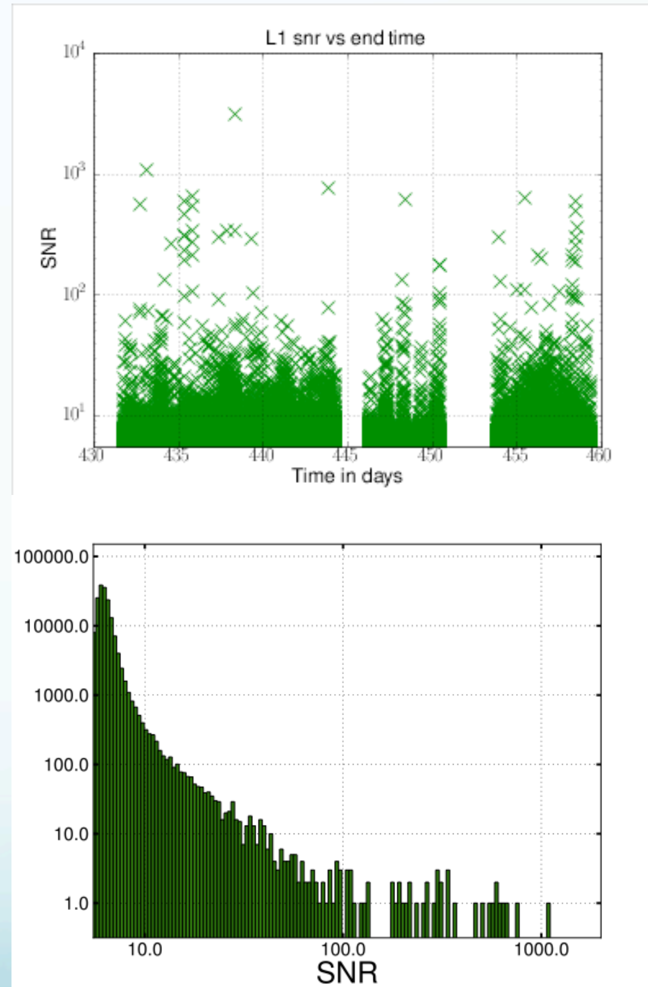


B. Abbott *et al*., *Rep. Prog. Phys.* **72** 076901 (2009)

- Noise distribution is strongly non-ideal at mid/low frequencies

Matched filter in non-Gaussian noise

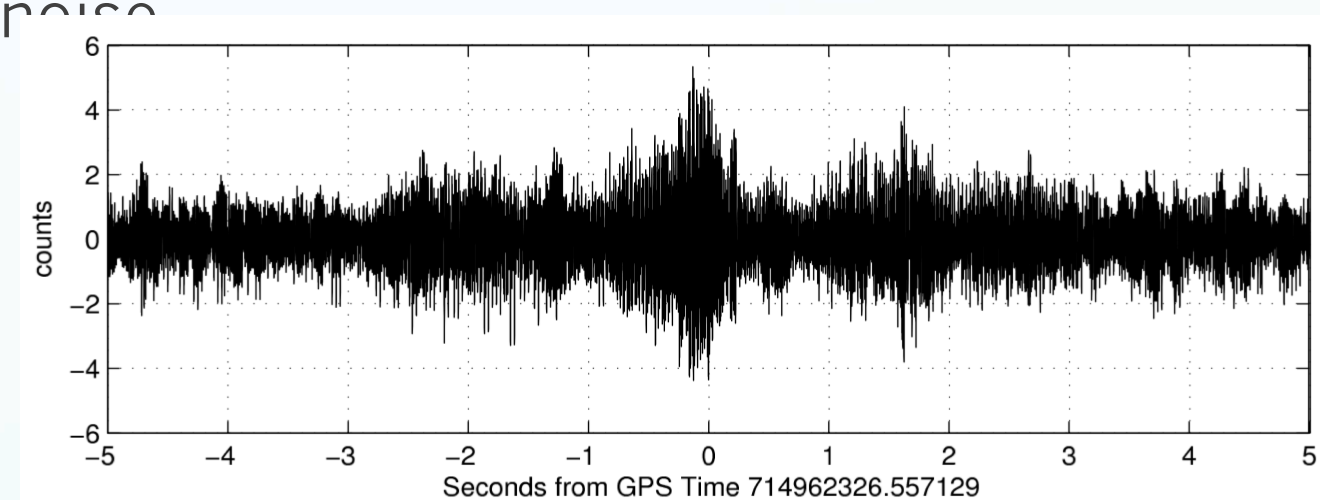




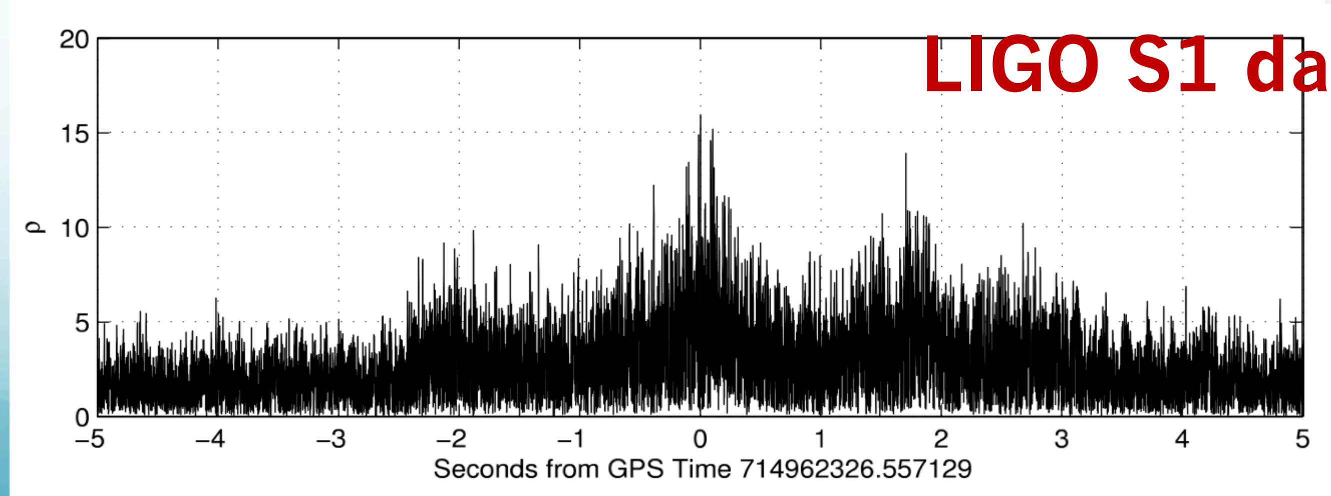
Non-Gaussian noise transients

- SNR above ~8 is not expected to occur ‘ever’ in Gaussian noise

Detector output



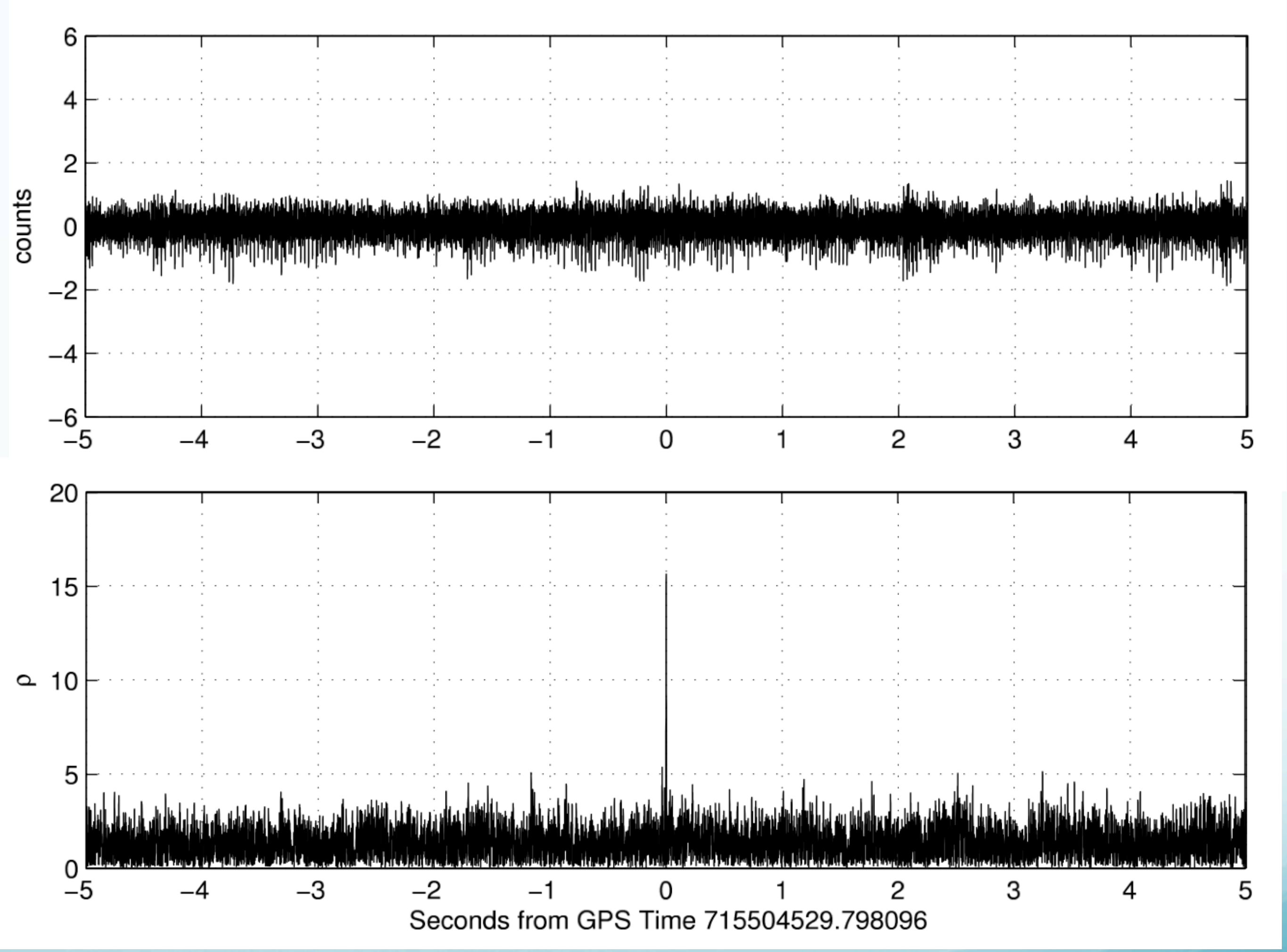
Matched filter ρ



Simulated signal in real noise

Detector output

Matched filter



‘Non-Gaussian noise’ questions

- What is it ?
  - could be almost anything ..
  - NEED TO **LOOK AT THE DATA**
- What causes it ?
  - Instrumental problems / environmental influence on detectors
  - In some cases: we do not know!
- What to do about it ?
  - SNR no longer ‘optimal’ even for single template
  - Need **to identify & use more information**

    – from detectors **and** in strain data